# WORKSHEET I

## REVIEW

1. Find the *area* of the region bounded by the x-axis and the curve

$$y = x(x - 1)(x - 3)$$

*Sketch!*

2. Find the area under one arch of the curve $y = \sin 4x$ *Sketch!*

3. Find the area bounded between the curves $y = x^2/2$ and $y = x + 4$.

4. Evaluate by first interpreting as area:

$$\int_0^3 \sqrt{9 - x^2}\, dx$$

5. Evaluate

$$\int_{-1}^1 x^3 \sqrt{5 + x^2}\, dx$$

(*Hint:* Think about the area interpretation of this integral.)

6. Evaluate

$$\int_{-1}^1 |\,3x + 1\,|\, dx$$

*Sketch!*

7. Show that

$$450 > \int_1^3 x^3\sqrt{1+x^6}\ dx > 300$$

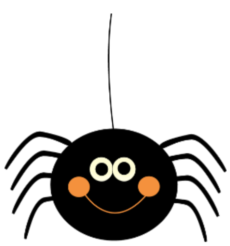
8. Using the *Fundamental Theorem of Calculus*, compute the derivative of the function

$$F(x) = \int_1^x \sin(t^2)\ dt$$

9. Suppose that Charlotte, the spider, travels along the x-axis from time t = 0 until t = 3 hrs and that her velocity function is given by:

$v(t) = t(1+t^2)^{1/2}$ mph.

*How far* does Charlotte travel during these three hours?



10. Using the method of *judicious guessing* or *substitution*, evaluate each of the following indefinite integrals:

(a) $\int \tan(4x)\sec^2(4x)\ dx$

(b) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(c) $\displaystyle\int \frac{x^2-5}{x+2}\,dx$

(d) $\displaystyle\int \frac{\sqrt{\ln x}}{x}\,dx$

11. Find the *maximum* value of the function $G(x) = -x^4 \ln x$ .

12. *Sketch* the curve below, finding all zeroes, singularities, horizontal and vertical asymptotes.

$$y = \frac{(x-1)^2(2x-3)^3}{(x+1)(x-2)^4}$$

13. *Sketch* the following curve, finding all *local extrema* and *points of inflection*. Where is the function *concave up*? *concave down*? Find *global extrema* if they exist.

$$y = x^3 e^{-2x}$$

14. Compute the following limit:

$$\lim_{x\to 0} \frac{e^x - x - 1}{\cos x - 1}$$

15. Find the point on the line x/a + y/b = 1 that is *closest* to the origin.

16. Find the values of *p* and *q* for which the function

$$F(x) = x^3 + px^2 + qx$$

(a) has a *local max* at x = -1 and a *local min* at x = 3.

(b) has a *local min* at x = 4 and a *point of inflection* at x = 1.

17. Express the following as a Riemann integral and evaluate:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^{100}$$

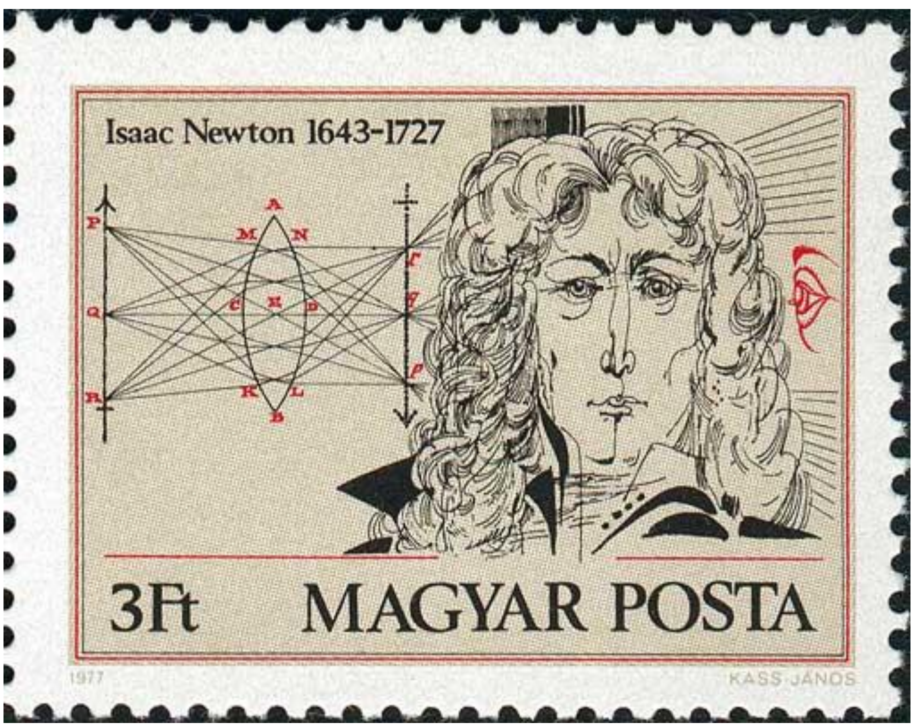
18. Give the definitions of the hyperbolic functions sinh x, cosh x, tanh x and sech x. Prove that $(\cosh x)^2 - (\sinh x)^2 = 1$ and $1 - (\tanh x)^2 = (\text{sech } x)^2$.

19. Find the *area* of the region bounded by the curves $y = x^{2016}$ and $y = x^{2015}$. Sketch!

20. *Find* $\lim_{x\to\infty} \dfrac{3(2x-5)^4(x^2-4x+2015)^2(3x+5)}{(x+11)^5(x+99)^2(5x+1)^2}$

*Twice and thrice over, as they say, good is it to repeat and review what is good.*

– Plato




-