WORKSHEET XII

NUMERICAL SERIES, *PART I*

1. Review: Explain why an increasing bounded *sequence* always converges. What is the corresponding result for decreasing sequences?

2. Explain precisely what it means for a series $\Sigma\, a_n$ to *converge*. What does it mean to say that a series *diverges*? What is meant by the *sum* of a series? In what sense is a series a particular type of sequence? Why is it important to distinguish between the *sequence of partial sums* and the *sequence of "atoms"*?

3. Discuss the rules for convergence (and divergence) of a sum, difference, or constant multiple of a series.

4. What characterizes a *geometric series*? When is a geometric series convergent? How would one find the sum of a convergent geometric series?

5. For each of the following series, $\sum a_n$, determine *convergence* or *divergence*. If the series converges, are you able to find its sum?

(a) $\sum (-1)^n$

(b) $\sum\ 3 / 4^n$

(c) $\sum\ 4^n / 3^n$

(d) $\sum\ 3/4^n$

(e) $\sum 1/n$

(f) $\sum \text{arc tan}\,(n)$

(g) $\sum e^{-n}$

(h) $\sum (1/2^n + 1/3^n)$

(i) $\sum (-1)^n / 7^n$

(j) $\sum (5)^{n+1} / 3^{2n+3}$

(k) $\sum (1+n) / (1+n^3)$

(l) $\sum (1+n^3) / (1+n^2)^2$

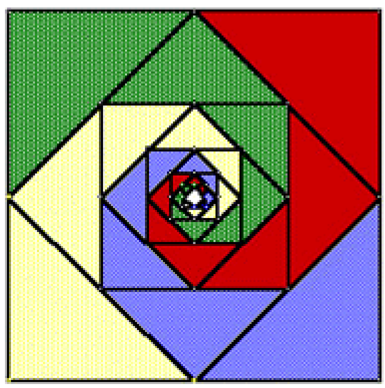
(m) $\sum$ (ln n) / (ln ln n)

(n) $\sum$ (cosh n) / (sinh n)

6. State the *n^{th} term test for divergence*.

7. State the *Comparison Test* for positive series.

8. How does this *Baravelle Spiral* represent an infinite series?



The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever. By using them, one may draw any conclusion he pleases and that is why these series have produced so many fallacies and so many paradoxes.

- Niels Henrik Abel