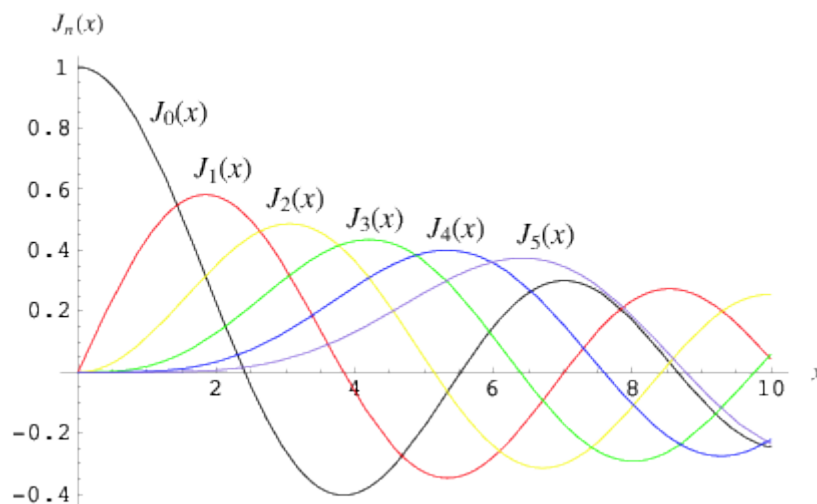


WORKSHEET XV

POWER SERIES



Bessel functions may be expressed as power series

1. For each of the following power series, determine the *interval of convergence*. Consider end-point behavior as well.

(a) $\sum \frac{x^n}{n^3}$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2}$

(d) $\sum_{n=1}^{\infty} \frac{(x+5)^n}{(2n+1)}$

(e) $\sum_{n=1}^{\infty} n! x^n$

(f) $\sum_{n=1}^{\infty} \frac{3n+5}{2016n+1} (x-1)^n$

(g) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n (x+4)^n$

$$(h) \sum_{n=2}^{\infty} \frac{(x-1)^n}{n \ln n}$$

$$(i) \sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{5^n}$$

$$(j) \sum_{n=1}^{\infty} \frac{x^n}{n+5^n}$$

$$(k) \sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$$

$$(l) \sum_{n=1}^{\infty} \frac{n x^{2n}}{1+3^n}$$

2. [University of Michigan Final Exam question] Consider the following power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n(2n+1)} (x-3)^n$$

(a) For which values of x does the power series converge?

(b) For which values of x does the power series converge absolutely?

(c) For which values of x does the power series converge conditionally?

3. [University of Michigan Final Exam question] Consider the following power series

$$\sum_{n=1}^{\infty} \frac{1}{4^n n^3} (x-2)^n$$

Find the interval of convergence of the power series. Justify your answer.

4. [University of Michigan Final Exam question] Consider the following power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(n+1)}(x+1)^{2n}$$

- (a) At $x = -3$, does the series converge absolutely, conditionally or diverge?
- (b) Using just your answer in (a), state the *possible* values for the radius of convergence R . Justify.
- (c) Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(n+1)}(x+1)^{2n}$$



If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

- John von Neumann