Worksheet XVI

Operations on Power series

1. (a) Consider the power series f(x) = 1/(1 – x) = 1 + x + x2 + x3 + … What is its interval of convergence?

(b) Find the series for 

1. Using the power series obtained in 1(b) for determine the sum of the series 
2. The series

sin x = x – x3/3! + x5/5! – x7/7! + x9/9! – x11/11! + …

 converges to sin x for all real x.

1. Find the first six terms of a series for cos x. For which values should the series converge?
2. By replacing x by 2x in the series for sin x, find a series that converges to sin 2x for all *x*.
3. Using series multiplication, find a series that converges to 2 sin x cos x.

4. (a) On which interval does the series 1/(1 + t) = 1 – t + t2 – t3 + t4 – t5 +… converge?

 (b) Integrating both sides over the interval [0, x], find the first six terms of a series that converges to ln(1 + x).

1. Find a power series representation of 1/(1 + x2).
2. Find a power series representation of 1/(2 + x2).
3. Find a power series representation of x4/(2 + x2).
4. Find a power series representation of 1/(1 – x)2. Differentiate a well-known geometric sum.
5. The series ex = 1 + x + x2/2! + x3/3! + x4/4! +x5 /5! + … converges to ex for all real *x*.
6. Find a series for (d/dx) ex.
7. Find a series for
8. In the series for ex, replace x by –x to find a series expansion of e-x.
9. In (c), replace x by x2 to find a series expansion of 
10. (a) Beginning with the series for 1/(1 + x2), find a series expansion of arctan x.

(b) Find a series expansion for 

1. Using a series representation for sin 3x, find values of *r* and *s* for which



1. *[Stewart]* Using the power series for arctan x, prove that the following series converges to :



*I used to love mathematics for its own sake, and I still do, because it allows for no* *hypocrisy and no vagueness….*

**-** Stendhal, **The Life of Henri Brulard**