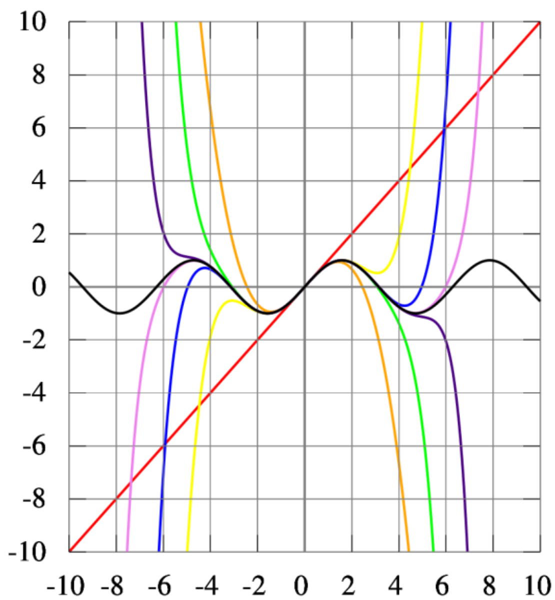
WORKSHEET XVII

TAYLOR POLYNOMIALS, TAYLOR SERIES



Wikipedia: As the degree of the Taylor polynomial rises, it approaches the correct function. This image shows sin(*x*) and its Taylor approximations, polynomials of degree 1, 3, 5, 7, 9, 11 and 13.

1. Find the 5^{th} order Maclaurin polynomial of e^{3x}.

2. Find the 4^{th} order Maclaurin polynomial of $(1 – x)\, e^x$.

3. Find the 3^{rd} order Taylor polynomial of $1/(1 + x^2)$ centered at $c = 1$.

4. Find the 5^{th} order Maclaurin polynomial of $(3x – \sin(3x))/x^3$.

5. Find the first four *non-zero* terms of the Maclaurin series of $\exp(x^2 + x)$.

6. Write the Maclaurin series expansion for $x/(1 + x^2)$ and for

$\ln(1 + x^2)$. Find the interval of convergence for each series. What is the relationship between these two series?

7. Using an appropriate power series expansion, compute $\Sigma\, n/7^n$. (*Hint:* Differentiate an appropriate geometric series.)

8. Find the Maclaurin series of each of the functions: $2/(3 – x)$, $5/(4 – x)$, and $(23 – 7x)/ [(3 – x)(4 – x)]$.

9. Find the 99th derivative of $1/(a – bx)$ by using an appropriate power series.

10. Find the *binomial expansion* of $(1 + x)^{-4}$. What is its radius of convergence?

11. Find the Maclaurin series expansion of $1/(1 + x^2)^{1/2}$.

12. Find the 23rd derivative of $1/(1 + x^2)^{1/2}$.

13. Using an appropriate Maclaurin series, evaluate the limit of $(\sin x – x)/x^3$ as $x \to 0$.

14. Evaluate the limit of $(\sin x – \tan x)/x^3$ as $x \to 0$ without using l'Hôpital's rule.

15. Evaluate the limit of $(\ln x) / (x – 1)$ as $x \to 1$ without using l'Hôpital's rule.

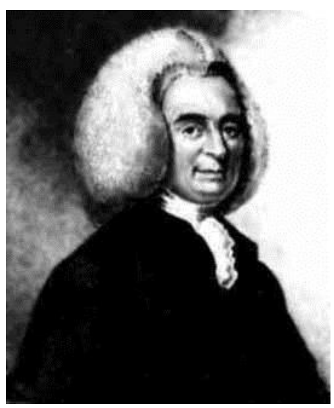
16. Evaluate the limit of $1/(\sin x) - 1/x$ as $x \to 0$ without using l'Hôpital's rule.

17. Evaluate the limit of $(\sin x - x)/(\tan x - x)$ as $x \to 0$ without using l'Hôpital's rule.

18. Evaluate the limit of $\ln x / (e^x - e)$ as $x \to 1$ without using l'Hôpital's rule. (Hint: Let $t = x - 1$.)

19. Find $\lim_{x\to 0} \dfrac{e^{x^2} - 1}{\cosh(3x) - 1}$ without using l'Hôpital's rule.

20. State Taylor's inequality. Using this inequality, prove that the Maclaurin series of e^x, $\sin x$, $\cos x$, and $\cosh x$ each converge to the given function everywhere.



Colin Maclaurin (1698 – 1746)

Brook Taylor (1685 - 1731)

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