

WORKSHEET XX

INTEGRATION OF TRIG FUNCTIONS

1. Integrate each of the following functions of sine x and cosine x :

(a) $\sin^2 4x$

(b) $\sin^3 x$

(c) $(\sin^2 x)(\cos^9 x)$

(d) $(\sin^3 x)\sqrt{1 + \cos x}$

(e) $\sin^4 3x$

(f) $\cos^5 x$

2. Integrate each of the following functions of secant x and tangent x :

(a) $\sec(4x)$

(b) $\sec^2 x$

(c) $\tan^2 x$

(d) $(\tan x)(\sec^2 x)$

(e) $(\tan x)(\sec^4 x)$

$$(f) (\tan^9 x)(\sec^2 x)$$

$$(g) (\tan^{10} x)(\sec^4 x)$$

$$(h) \sec^3 x$$

3. Integrate each of the following functions by completing the square, if necessary.

$$(a) \frac{1}{x^2 + 4x + 5}$$

$$(b) \frac{1}{x^2 + 2x + 5}$$

$$(c) \frac{1}{(x^2 + 1)^2}$$

$$(d) \frac{x}{x^2 + 4x + 11}$$

4. By making an appropriate trig (or hyperbolic) substitution, convert each of the following integrals to trig integrals.

$$(a) \int \frac{x}{(x^2 + 1)^3} dx$$

$$(b) \int \frac{x^3}{\sqrt{1 - x^2}} dx$$

$$(c) \int \frac{x^2}{\sqrt{x^2 - 1}} dx$$

$$(d) \int \frac{\sqrt{1-x^2}}{x^2} dx$$

5. Using an appropriate trig identity, evaluate each of the following trigonometric integrals:

$$(a) \int \cos(4x) \sin(8x) dx$$

$$(b) \int \cos(2x) \cos(5x) dx$$

*There was a young fellow called Dan,
Who knew all about sin, cos and tan.
He talked rather big
Of his knowledge of trig –
He did seem a clever young man.*

– Anonymous

