WORKSHEET XXI

PARTIAL FRACTION DECOMPOSITION

1. Find the *partial fraction decomposition* of each of the following rational functions. (To check your answers in Mathematica, use the *Apart* command.)

(a) $\dfrac{x+3}{(x+1)(x+2)}$

(b) $\dfrac{x^4+1}{x(x-2)}$

(c) $\dfrac{1}{(x+5)(x-2)(x-3)}$

(d) $\dfrac{x^2+1}{(x+4)^2(x-2)}$

(e) $\dfrac{x+3}{(x+5)(x-2)(x-3)}$

2. Find the *form* of the partial fraction decomposition of each of the following. You need not solve for the constants.

(a) $\dfrac{x+8}{(x+5)^3(x-2)^2(x-3)}$

(b) $\dfrac{3x}{(x^2+5)(x^2-7x+10)}$

(c) $\dfrac{x^5 + x + 1}{(x^2 + x + 5)^2 (x + 2016)}$

(d) $\dfrac{x^{11} + 4x^4 + 1}{(x^2 + 1)^3 (x - 1)(x + 2)^4}$

3. Compute the *indefinite integral* of each of the following functions.

(Warning: first convert to a rational function!)

(a) $\dfrac{1}{e^{2x} - 2e^x - 35}$

(b) $\dfrac{\sin x}{\cos^2 x + \cos x - 20}$

(c) $\dfrac{1}{x(\ln x)\left(1 + (\ln x)^2\right)}$

We live but a fraction of our lives.

- <u>Henry David Thoreau</u>

This is a tricky domain because, unlike simple arithmetic, to solve a calculus problem - and in particular to perform integration - you have to be smart about which integration technique should be used: integration by partial fractions, integration by parts, and so on.

- <u>Marvin Minsky</u>