

WORKSHEET XXII

INTEGRATION OF TRIG FUNCTIONS

1. Integrate each of the following functions of sine x and cosine x :

(a) $\sin^2 4x$

(b) $\sin^3 x$

(c) $(\sin^2 x)(\cos^9 x)$

(d) $(\sin^3 x)\sqrt{1+\cos x}$

(e) $\sin^4 3x$

(f) $\cos^5 x$

2. Integrate each of the following functions of secant x and tangent x :

(a) $\sec(4x)$

(b) $\sec^2 x$

(c) $\tan^2 x$

(d) $(\tan x)(\sec^2 x)$

(e) $(\tan x)(\sec^4 x)$

(f) $(\tan^9 x)(\sec^2 x)$

(g) $(\tan^{10} x)(\sec^4 x)$

(h) $\sec^3 x$

3. Integrate each of the following functions by completing the square, if necessary.

(a) $\frac{1}{x^2 + 4x + 5}$

(b) $\frac{1}{x^2 + 2x + 5}$

(c) $\frac{1}{(x^2 + 1)^2}$

(d) $\frac{x}{x^2 + 4x + 11}$

4. By making an appropriate trig (or hyperbolic) substitution, convert each of the following integrals to trig integrals.

(a) $\int \frac{x}{(x^2 + 1)^3} dx$

(b) $\int \frac{x^3}{\sqrt{1 - x^2}} dx$

(c) $\int \frac{x^2}{\sqrt{x^2 - 1}} dx$

$$(d) \int \frac{\sqrt{1-x^2}}{x^2} dx$$

5. Using an appropriate trig identity, evaluate each of the following trigonometric integrals:

$$(a) \int \cos(4x) \sin(8x) dx$$

$$(b) \int \cos(2x) \cos(5x) dx$$

$$(c) \int \sin(5x) \sin(9x) dx$$

