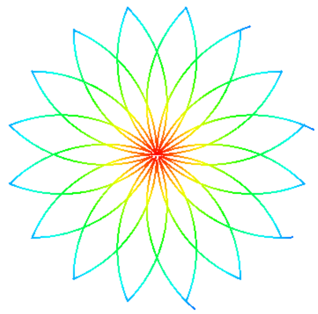
WORKSHEET XXIII

POLAR COORDINATES



1. (a) Convert from polar coordinates to Cartesian coordinates: (3, 0), (1, $\pi/4$), ($-2^{1/2}$, π /6), (4, $3\pi/2$), (7, $5\pi/3$)

(b) Convert from Cartesian coordinates to polar coordinates: (5, 5), (-3, 0), (1, $-3^{1/2}$), (-7, -11)

(c) Which polar coordinate pairs label the same point? (3, 0), (-3, 0), (2, $2\pi/3$), (2, $7\pi/3$), (-3, π), (2, $\pi/3$), (-3, 2π), (-2, $-\pi/3$)

2. Write each of the following polar equations as a Cartesian equation:

(a) $r \cos \theta = 2$

(b) $r \sin \theta = 0$

(c) $r \cos \theta = 0$

(d) $r (\cos \theta + \sin \theta) = 1$

(e) $r^2 = 4r \sin \theta$

(f) $r^2 \sin 2\theta = 2$

(g) $r = \dfrac{5}{\sin\theta - 2\cos\theta}$

(h) $r = 11$

3. Convert each Cartesian equation below to a polar equation.

(a) $x = 7$

(b) $x^2 + y^2 = 4$

(c) $x^2 - y^2 = 1$

(d) $xy = 2$

(e) $x^2 + xy + y^2 = 1$

(f) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

4. In sketching a polar curve how would one check for symmetry (a) about the origin? (b) about the x-axis? (c) about the y-axis?

5. Sketch the following polar curves:

(a) $r = 3$

(b) $\theta = \pi/3$, $-1 \le r \le 3$

(c) $r = -1$, $0 \le \theta \le \pi$

(d) $r = \theta$ (*spiral of Archimedes*)

(e) $r = 1 - \cos\theta$ *(cardioid)*

(f) $r = 6 \sin\theta$

(g) $r\theta = 1$ *(hyperbolic spiral)*

(h) $r = 1 + 2\sin\theta$ *(looped limaçon)*

(i) $r = 3 + 2\sin\theta$ *(dimpled limaçon)*

(j) $r = \cos 2\theta$ *(rose)*

(k) $r = \cos 3\theta$ *(rose)*

(l) $r = \cos 4\theta$ *(rose)*

(m) $r = e^{\theta}$ *(logarithmic spiral)*

(n) $r^2 = \theta$ *(Fermat's spiral)*

(o) $r^2 = \cos 2\theta$ *(lemniscate of Bernoulli)*

6. Derive a formula for the area of the fan-shaped region between the origin and the curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$.

Find the area of the region:

(a) bounded by the spiral $r = \theta$ for $0 \leq \theta \leq \pi$

(b) enclosed by the cardioid $r = 2(1 + \cos \theta)$

(c) inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$

(d) enclosed by the smaller loop of the limaçon $r = 2 \cos \theta + 1$

(e) enclosed by one leaf of the four-leaved rose $r = \cos 2\theta$

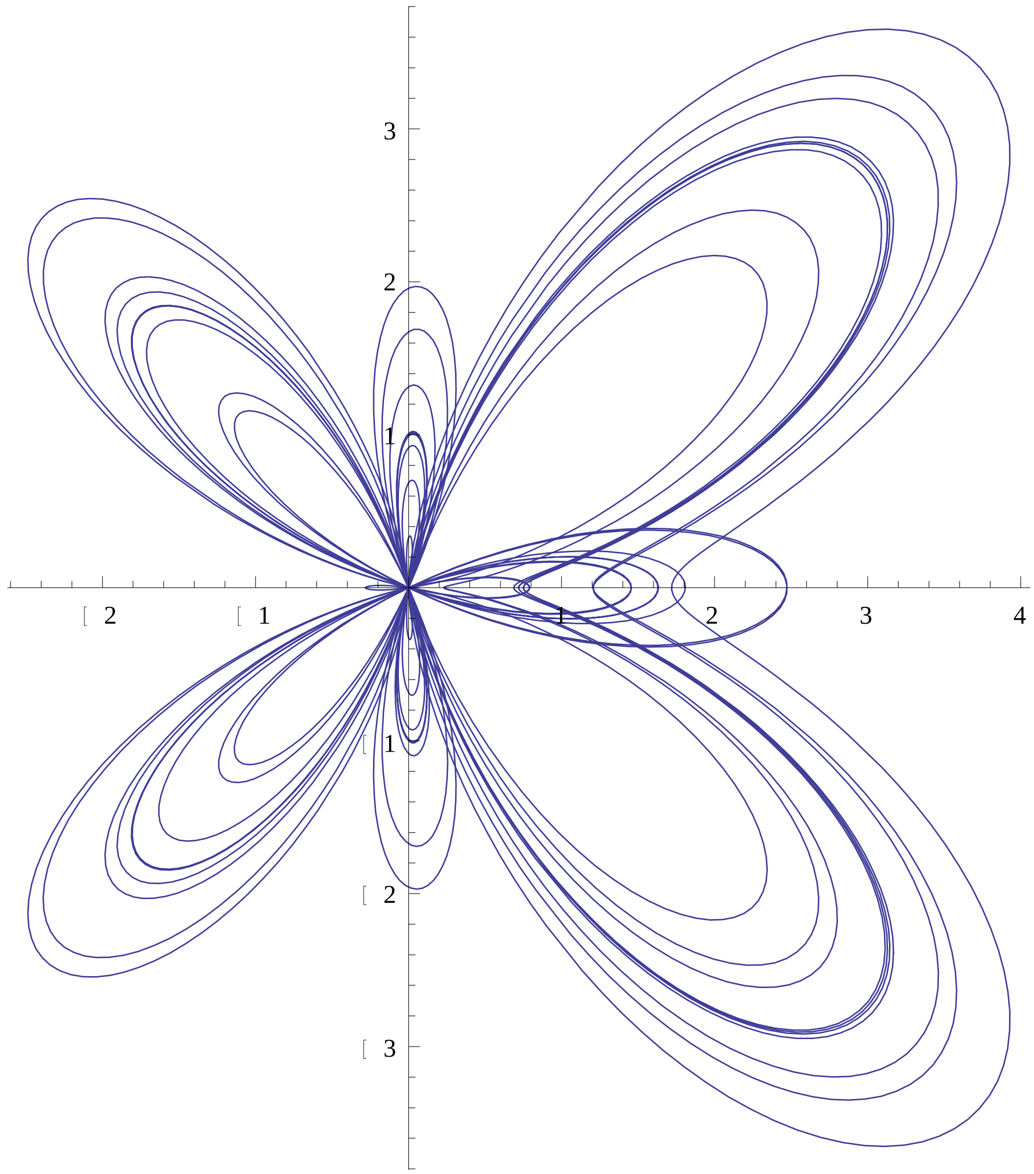
7. Derive a formula for the arc length of a curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$. Find the arc length of the

(a) circle $r = b$

(b) circle $r = a \cos \theta$, $-\pi/2 \leq \theta \leq \pi/2$

(c) spiral $r = \theta^2$, $0 \leq \theta \leq \sqrt{5}$

(d) cardioid $r = 1 - \cos \theta$



Mathematica polar plot of r = $e^{\cos\theta} - 2\cos(4\theta) + \sin^5(\theta/12)$ for $0 \le \theta \le 20\pi$