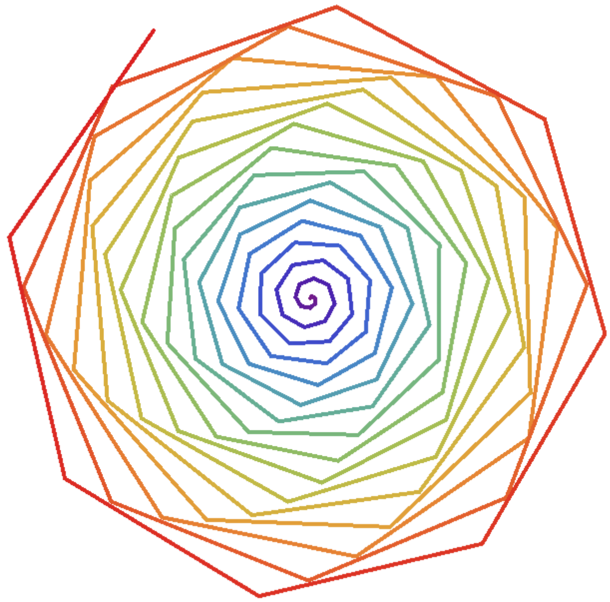
WORKSHEET IV (REVISED)

PARAMETRIC EQUATIONS – A BRIEF INTRODUCTION



1. Sketch the curve $x(t) = 3t$, $y(t) = t^2 + 1$. Express *y* as a function of *x*.

2. Describe the parameterized curve $x(t) = 3 \cos t$, $y(t) = 4 \cos t$, $0 \leq t \leq 2\pi$.

   What is the relationship between the given curve above and each of the following?

   (a) $x(t) = -3 \cos t$, $y(t) = 4 \cos t$, $0 \leq t \leq 2\pi$.

   (b) $x(t) = 3 \cos 2t$, $y(t) = 4 \cos 2t$, $0 \leq t \leq 2\pi$.

   (c) $x(t) = 1 – 3 \cos 2t$, $y(t) = 1 – 4 \cos 2t$, $0 \leq t \leq 2\pi$.

3. Show that the following is a parameterization of the cycloid:

   $$x(\theta) = a(\theta – \sin \theta),\ y(\theta) = a(1 – \cos \theta),\quad -\infty < \theta < \infty.$$

4. Show that $x = a \cos t + h$, $y = b \sin t + k$, $0 \leq t \leq 2\pi$, is a parametric equation of an ellipse with center at (h, k) and axes of length $2a$ and $2b$.

5. Find a parameterization of the straight line $y = 3x + 4$.

6. Find a parameterization of the straight line segment joining the points $P = (3, 5)$ to $Q = (7, 11)$.

7. Find a parameterization of the curve $y = x^2$ from

P = (-1, 1) to Q = (4, 16).

8. Generalize problem 7 for any curve of the form $y = f(x)$ from

x = a to x = b.

9. Find an equation of a line tangent to the given curve at the given point.

(a) x = sin 2pt, y = cos 2pt, t = -1/6

(b) x = 1/t, y = -2 + ln t, t = 1

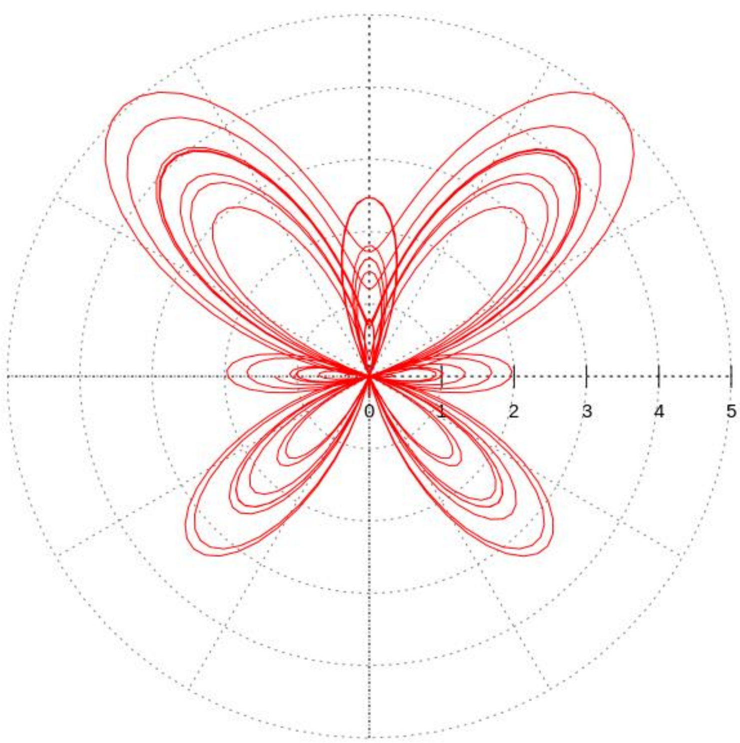
(c) x = t – sin t, y = 1 – cos t, t = p/3

(d) $x = t + e^t$, $y = 1 - e^t$, t = 0.

10. Find d^2y/dx^2 as a function of time if $x = t - t^2$ and $y = t - t^3$.

11. Find an equation for the line in the xy-plane that is tangent to the curve

X = ½ tan t, y = ½ sect t, at t = p/3. Also find d^2y/dx^2 at the given point.



the butterfly curve

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