

# WORKSHEET VIII

## LITTLE OH AND BIG OH

Suppose that  $f(x) \rightarrow \infty$  and  $g(x) \rightarrow \infty$  as  $x \rightarrow \infty$ . We say that “ $f$  is of smaller order than  $g$ ” if  $\frac{f(x)}{g(x)} \rightarrow 0$  as  $x \rightarrow \infty$ . In this case we write  $f = o(g)$ .

Assume that  $f$  and  $g$  are each positive for large  $x$ . We say that “ $f$  is at most the order of  $g$ ” if there is a positive integer  $M$  for which  $\frac{f(x)}{g(x)} \leq M$  for large  $x$ . In this case we write  $f = O(g)$ .

Determine which of the following statements are true; justify each answer.

(a)  $3x^2 + 11 = o(x^5 + x + 99)$

(b)  $x + 5 \sin x = O(x)$

(c)  $2^x = o(x^{100})$

(d)  $3^x = O(e^x)$

(e)  $x = o(\ln x)$

(f)  $3 + \ln x + \ln(\ln x) + \sqrt{x} = o\left(x^{\frac{2}{3}}\right)$

(g)  $\ln x = o(\sqrt{x})$

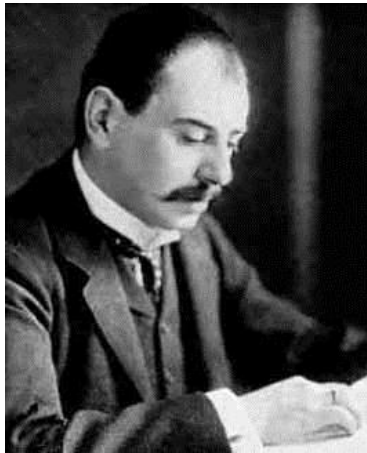
(h)  $(x^2+1)^4 = O((2x+1)^3x^5)$

(i)  $\frac{x^2 + 13x + 2016}{5x + 1789} = O(\sqrt{x^2 + 9})$

(j)  $\ln x = o(\ln(\ln x))$

(k)  $\ln(x^{55} + x^{33} + x^{11} + 1) = O(\ln x)$

(l)  $(\ln x)^{100} = o(x^{1/25})$



[Edmund Landau](#) (1877 – 1938) is known for his work in analytic number theory and the distribution of primes. He first introduced the *little oh* notation in 1909.