

MATHEMATICA LAB III

TAYLOR SERIES

(Due: 18 April 2018)

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I Power Series

1. Find the 8th order Maclaurin polynomial of $\tanh(x)$.
2. Find the 7th order Taylor polynomial of e^x about $x = 1$.
3. Find the 9th order Maclaurin polynomial of $\ln(1+x)$.
4. Plot the graph of $y = e^x$ along with the first four Maclaurin polynomials of e^x on the same set of axes.
5. Find the 14th order Maclaurin polynomial of $\exp(x^2)$. Can you see how this polynomial is related to the 7th order Maclaurin polynomial of e^x ? Explain.

II Weierstrass' example



Weierstrass

Here we examine a function defined by an infinite series (that is *not* a power series) which is continuous but nowhere differentiable.

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \sin(3^n x)$$

6. Plot the n^{th} partial sum of $f(x)$ for *several values* of n (for example, $n = 3, 5, 8$). Why might you believe that $f(x)$ is not differentiable?

III Infinite products

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots = \prod_{n=1}^{\infty} \left(\frac{4n^2}{4n^2 - 1} \right).$$

Formula of [John Wallis](#) (1616 – 1703)



In mathematics, **infinite products** play an important role, although perhaps not quite as prominent a role as that of infinite series. Analogous to infinite series, an infinite product is the limit of a sequence of *partial products*. The capital Greek letter, Pi, is used to indicate a product. For example,

$$\prod_{k=1}^n a(k)$$

denotes the product: $a(1)a(2)a(3)\dots a(n)$. If we wish to define an *infinite product*, we could let $p(n) = a(1)a(2)a(3)\dots a(n)$ and define the infinite product to equal the limit of $p(n)$ as n increases without bound, if the limit exists. Of course, if the limit does not exist, we say that the infinite product *diverges*.

7. Examine the infinite products defined by $a(n) = 1 + 1/n$ and $b(n) = 1 + 1/n^2$. Graph each sequence of partial products. Does either infinite product converge? If so, what is its limit?

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