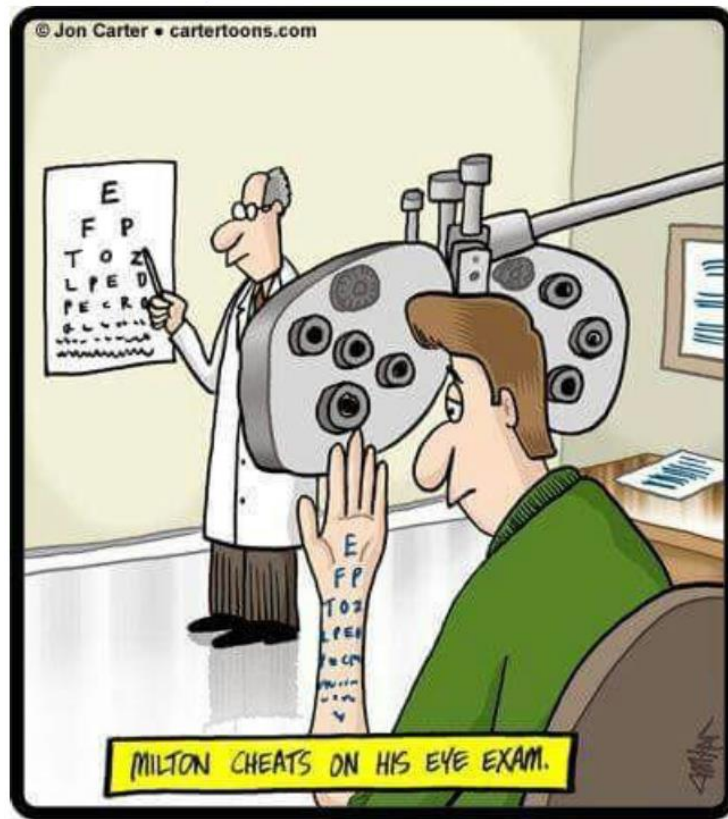


MATH 162 PRACTICE FINAL EXAM A



Answer any 19 of the 22 questions. You may answer more than 19 to obtain extra credit. For each of the following questions, justify your answer! You will receive little or no credit for simply writing a correct answer.

1.
$$\lim_{n \rightarrow \infty} \left(n \tan \frac{3}{n} + \frac{\ln n}{\sqrt{n}} + \frac{2013^n}{n!} + \left(1 + \frac{5}{n}\right)^n - 4 \frac{(1+n)^3 (1+2n)^3}{(5+n)^2 (7+n)^4} \right) =$$

2. Find the *first five* non-zero terms of the Maclaurin series expansion of

$$h(x) = (1 + 2x^2)e^{3x}.$$

3. Determine the *interval of convergence* of the following power series. (You need not study end-point behavior.)

$$\sum_{n=1}^{\infty} \frac{n^5 7^n}{\sqrt{n+2007}} (x+4)^n$$

4. Determine *convergence* or *divergence* of the series
$$\sum_{n=1}^{\infty} \frac{((2n)!)^2}{(n!)^3}$$

5. Find the *area* between the following curve and the x-axis from $x = 0$ to $x = 1$.

$$y = \frac{x+1}{\sqrt{4-x^2}}$$

Simplify your answer as much as possible.

6. Using the integration theorem for convergent power series, compute the Maclaurin series of the function $g(x) = \ln(1 - 2x)$. Include the general expression for the n^{th} term of the series. *Hint:* Begin by writing $1/(1-2x)$ as a familiar power series.

7. Find explicitly the n^{th} partial sum of the series $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$ and thereby determine whether or not the series converges.

8. Does the following improper integral *converge* or *diverge*? Explain!

$$\int_3^{\infty} \frac{(1+e^x)(2+e^{3x})+x^5+11x+13}{x(\ln x)^2(4+e^x)^4} dx$$

9. Using constants A, B, C , etc, determine the correct form of the partial fraction decomposition of the following rational function. Do *not solve* for the values of A, B, C, \dots

$$\frac{7x^3 + 4x + 9}{(x-3)^3(x-4)(x^2+11)^2}$$

10. Does the following improper integral *converge* or *diverge*? Explain fully!

$$\int_{0^+}^{\infty} \frac{1}{(x+x^9)^{\frac{1}{3}}} dx$$

11. Evaluate the integral $\int x^2 \arctan x dx$

12. Using only the definition of $\sinh x$ and employing basic algebra, prove that

$$\sinh^3 x = 3 \sinh x + 4 \sinh^3 x$$

13. The area bounded by the curve $y = \frac{x}{\sqrt{x^3+8}}$, the x-axis, and the line $x = 2$ is rotated

about the y-axis generating a certain volume. Set up the integral that should be used to evaluate the volume and then evaluate the integral.

14. The base of a solid is the area bounded by $y^2 = 8x$ and $x = 2$. Each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of the solid.

15. Sum the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$. Be certain to explain each step of your work!

16. A chlorine solution is poured over the surface of Albertine's rectangular swimming pool that is 40 meters long, 30 meters wide, and 3 meters deep everywhere. Before the circulating pumps are turned on, it is discovered that the density of the chlorine solution at a height h meters above the bottom of the pool is given by $\rho(h) = 80(10 - h^2)$ gm/m³. (That is, the chlorine solution's density is greater near the bottom of the pool.)

- Express the total mass of chlorine in Albertine's pool as a Riemann sum.
- Transform the Riemann sum of part (a) into a definite integral that gives the total mass of the chlorine solution in Albertine's pool. (You need not evaluate this definite integral.)

17. The arc of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ from $x = 1$ to $x = 2$ is rotated about the x -axis.

Calculate the area of this surface of revolution. Simplify your answer as much as possible.

18. (a) Using Euler's identity, find the three cube roots of i .

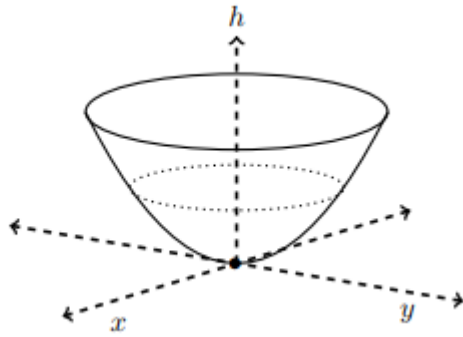
(b) Using Euler's identity, find a formula for $\sin(5x)$.

19. Consider the sequence $a_n = ((1/3)^n + (1/5)^n)^{1/n}$. Prove that this sequence converges and find its limit.

20. Solve the equation $z^2 = -i$.

21. Express in the form $a + bi$: $\frac{(2+3i)(4-2i)}{(1+i)(2+i)}$

22. Albertine is having a party to celebrate the end of the semester at Loyola; she is serving punch out of a parabolic punch bowl. The bowl is resting on a table (the xy -plane) as depicted in the figure below. At a height h above the table, the cross section of the bowl perpendicular to the h -axis is a circle with equation, $h = 4x^2 + 4y^2$.



The punch bowl is 1 meter tall. Assume the units of x , y , and h are in meters and the density of the punch is 1200 kg/m^3 . Recall the gravitational constant is $g = 9.8 \text{ m/sec}^2$.

- Write an expression for the mass of a slice of punch of thickness Δh meters at a height h meters above the table.
- Assuming the bowl is filled with punch up to a height of $h = 1/2$, write an integral which gives the amount of work needed to lift all of the punch over the rim of the bowl. Do not evaluate your integral.

Whoever despises the high wisdom of mathematics nourishes himself on delusion and will never still the sophistic sciences whose only product is an eternal uproar.

Mechanics is the paradise of the mathematical sciences, because by means of it one comes to the fruits of mathematics.

He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast.

No human investigation can be called real science if it cannot be demonstrated mathematically.

- Leonardo da Vinci (1452-1519)