

# MATH 162 PRACTICE FINAL EXAMINATION B



*Ed totally blows the final portion of his  
Chameleon Aptitude Test.*

*Answer any 30 of the following 35 questions. You may answer more than 30 to obtain extra credit. In general, show your work.*

1. Give the form of the partial fraction decomposition of  $\frac{1}{x^3(x^2 + 13)^2}$ . Do not solve for constants A, B, C, etc.
2. Evaluate the improper integral  $\int_2^{\infty} \frac{1}{x(\ln x)^3} dx$ .
3. Evaluate  $\int \arccos x dx$ .

4. Consider the sequence

$$a_n = \frac{\ln n}{\ln 2n},$$

defined for  $n \geq 1$ . Does this sequence converge or diverge? In the case of convergence, find its limit.

5. Find the equation of a curve in the  $xy$ -plane that passes through the origin and whose arc length from  $x = 0$  to  $x = 1$  is given by:

$$s = \int_0^1 \sqrt{1 + \frac{1}{4}e^x} dx$$

6. Consider the sequence

$$b_n = n \sin\left(\frac{5}{n}\right),$$

defined for  $n \geq 1$ . Does this sequence converge or diverge? In the case of convergence, find its limit.

7. Consider the series

$$\sum_0^{\infty} (-1)^n \frac{1}{1 + \sqrt{n}}$$

Does this series converge absolutely, converge conditionally, or diverge? Explain briefly.

8. Sum the series

$$\sum_0^{\infty} (-1)^n \frac{3}{2^n}$$

9. Find the first four *non-zero* terms of the Maclaurin series expansion of

$$x^4 \cos(x^2).$$

10. Evaluate the improper integral

$$\int_0^{\infty} \frac{32 \arctan x}{1+x^2} dx$$

11. Set up an integral for the area of the surface generated by revolving  $y = \tan x$ ,  $0 \leq x \leq \pi/4$ , about the  $x$ -axis. *Do not evaluate.* Sketch.

12. Express as a trigonometric integral. *Do not evaluate.*

$$\int x^{13} (4-x^2)^{\frac{9}{2}} dx$$

13. Consider the sequence

$$c_n = \left( \frac{e^n}{n!} \right)^2 + \frac{n^2 + 1}{(n+1)^2}$$

defined for  $n \geq 1$ . Does this sequence converge or diverge? In the case of convergence, find its limit.

14. Evaluate  $\int \frac{e^x}{e^{2x} + e^x - 12} dx$

15. Find the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)} (x-1)^n$$

You need not test the endpoints for convergence.

18. Evaluate

$$\int \sec \theta \tan \theta \sqrt{1 + \sec \theta} d\theta.$$

19. Find the volume of the solid generated by revolving the region bounded by

$$y = \sqrt{9 - x^2} \text{ and } y = 0$$

about the x-axis. (You may use any method you wish.) Sketch.

20. Albertine, a mountain climber, is about to haul up a 50 m. length of hanging rope. How much work will she do if the rope weighs 0.625 N/m?

21. Evaluate

$$\int e^x \sec^2(e^x - 13) dx.$$

22. The region enclosed by the x-axis and the parabola  $y = 3x - x^2$  is rotated about the vertical line  $x = -1$ . Using shells, write an integral that represents the volume of this solid. Sketch. *Do not* evaluate.

23. Does the following improper integral converge or diverge? Explain.

$$\int_{0+}^{\pi} \frac{dt}{\sqrt{t} + \sin t}.$$

24. Does the following improper integral converge or diverge? Explain.

$$\int_1^{\infty} \frac{x(x^2 + 1)^2 (x^3 + 1)^3}{(x^4 + 1)^4} dx.$$

25. *Without using l'Hôpital's rule*, calculate:

$$\lim_{t \rightarrow 0} \frac{1 - \cos t - \frac{t^2}{2}}{t^4}.$$

26. Does the following series converge or diverge? Explain.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

27. Does the following series converge or diverge? Explain.

$$\sum_1^{\infty} \frac{n + \cos^2 n}{n^3 + n^2 + n + 1}$$

28. Consider the following recursively defined sequence:

$$a_1 = 1, a_2 = 4, \text{ and } a_n = \frac{a_{n-1} + a_{n-2} + 1}{3} \text{ for } n \geq 3.$$

Find  $a_5$ .

29. Find the interval of convergence of the power series:

$$\sum_1^{\infty} \frac{1}{3^n + 4^n} (x + 11)^n$$

You need not test the endpoints for convergence.

30. The following probability density function

$$f(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{111} e^{-\frac{t}{111}} & \text{for } t \geq 0 \end{cases}$$

models the length of time that a 60 watt bulb manufactured by Alphaville Electrical will last. Compute the probability that a random 60 watt bulb purchased from Alphaville Electrical will die within the first 25 hours of use. Express your answer to the nearest hundredth.

31. Let  $f(x) = x^8 e^{2x}$ . Compute  $f^{(100)}(0)$ . Do not simplify your answer.
32. Does the following series converge or diverge? Explain.

$$\sum_{n=1}^{\infty} \sqrt{e^{\frac{n}{n^2+1}}}$$

33. Find the sum of the following series:

$$\sum_{n=1}^{\infty} \left( \frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)} \right)$$

35. Evaluate  $\int \frac{t^3 + t + 4}{t^2 - 9} dt$ .

36. Solve the differential equation:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$$

**Additional Extra Credit:**

Evaluate the series without using a calculator!

$$\frac{(1)(2)(3)}{4} + \frac{(2)(3)(4)}{4^2} + \frac{(3)(4)(5)}{4^3} + \frac{(4)(5)(6)}{4^4} + \dots$$