MATH 162 PRACTICE TEST 1

1. Determine *convergence* or *divergence* of each of the following improper integrals:

$$(a) \quad \int_{0+}^{\frac{1}{e}} \frac{(-\ln x)}{x^4}\, dx$$

$$(b) \quad \int_{0+}^{\infty} \frac{1}{\sqrt{x+x^4}}\, dx$$

$$(c) \quad \int_{0+}^{\infty} \frac{5x+7}{\sqrt{x^2+3x^4}}\, dx$$

3. *Evaluate* each of the following convergent improper integrals. Show your work!

(A) $$\int_{0}^{\infty} t^3 e^{-t^4}\, dt$$

(B) $$\int_{3}^{\infty} \frac{1}{x(1+\ln x)^{7/3}}\, dt$$

4. For each of the following improper integrals, determine convergence or divergence. *Justify each answer! (That is, if you use the comparison test, exhibit the function that you choose to use for comparison and show why the appropriate inequality holds.)*

(A) $$\int_{0}^{\infty} \frac{1+x+x^4}{(1+x)^5}\, dx$$

(B) $$\int_0^\infty \frac{1+x+e^x}{5+3e^{3x}}\,dx$$

5. For each given sequence, determine *convergence* or *divergence*. Justify your answers.

(a) $$a_n = \frac{100^n + 1789^n}{n! + 7^n}$$

(b) $$b_n = \left(1 + \frac{1}{3n}\right)^n$$

(c) $$c_n = \frac{\ln(n + 2018\pi)}{\ln n}$$

(d) $$d_n = \frac{\cos\left(\frac{\pi}{n}\right)}{n}$$

(e) $$e_n = \int_0^n e^{-\pi t}\,dt$$

(f) $$f_n = \sqrt{\frac{n+1}{n} + \frac{\sin(n^2)}{n^2} + e^{-\frac{3}{n}} + \frac{e^n}{\sinh n}}$$

(g) $$g_n = \frac{(\ln\ln n)^{2525}}{n}$$

6. Give an example of two *divergent* numerical sequences whose sum is *convergent.*

9. Consider the following recursively defined sequence:

$c_1 = 7$, $c_2 = 4$, and

$$c_{n+1} = \frac{\left(c_n + \dfrac{5}{(c_{n-1})^2}\right)}{2} \qquad for\ \ n \geq 2$$

(a) Find the values of c_3, c_4 and c_5.

(b) Assuming that the limit of c_n (as $n \to \infty$) exists, find its value.

10. *Find* $\lim_{n\to\infty} n^{\frac{1}{\ln n}}$ *(Show your work!)*

12. For each of the following improper integrals, determine convergence or divergence.

$$\int_1^\infty \frac{2+\sin x}{\sqrt{x+1}}dx$$

$$\int_1^\infty \frac{\theta}{\sqrt{\theta^5+1}}d\theta$$

$$\int_0^1 \ln(x)\,dx$$

$$\int_2^\infty \frac{x+\sin x}{x^2-x}\,dx$$

13. A machine produces copper wire, and occasionally there is a flaw at some point along the wire. The length x of the wire produced between two consecutive flaws is a continuous random variable with probability density function:

$$f(x) = \begin{cases} c(1+x)^{-3} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Show your work in order to receive full credit:

(a) Find the value of *c*.

(b) Find the *cumulative distribution function* P(x) of the density function f(x). Remember that a cumulative distribution function is defined on the entire real line.

(c) Find the *expected value* (aka mean) of the length of wire between two consecutive flaws.

5. *For each of the following sequences, determine convergence or divergence. In the case of convergence, find the limit of the sequence.*

(a) $x_n = e^{\frac{1}{n}}$

(b) $y_n = \dfrac{n!}{n+1}$

(c) $z_n = \dfrac{\sin n}{n} + \dfrac{5}{n}$

(d) $c_n = \dfrac{3(2n+1)^3}{(1-n)^2(4n+13)}$

8. For each improper integral given below, determine *convergence* or *divergence*. (You will need to use the Comparison Test here.) *Justify your answers!*

(a) $\int_0^\infty \frac{sin^{2018}(3x+5)}{(2018+x)^2}\ dx$

(b) $$\int_4^{\infty} \frac{1}{(\ln x)-1}\, dx$$

(c) $$\int_0^{\infty} \frac{(3+x)^2+133x\ln x+5x+1}{(1+99x+x^2)^4}\, dx$$

(d) $$\int_1^{\infty} \frac{\ln x}{x^3}\, dx$$

(e) $$a_n = \sec\left(\ln\left(\sin^4\left(\frac{\pi}{2}+\frac{1}{n^2}\right)\right)\right)$$

11. In March of 2015 it was announced that, "after six years of planetary observations, scientists at NASA say they have found convincing new evidence that ancient Mars had an ocean." According to the New York Times report, "it was probably the size of the Arctic Ocean", larger than previously estimated, the researchers reported on Thursday. "The body of water spread across the low-lying plain of the planet's northern hemisphere for millions of years", they said.

Professor Albertine at Caltech has developed a new theory about these ancient sea beds of Mars. She believes that 3.8 billion years ago there was a huge population of eels living in this Martian ocean. In her model, the length of an eel is given by the probability density function

$$f(x) = \begin{cases} kxe^{-x^2} & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases}$$

where x is measured in meters and k is a positive constant.

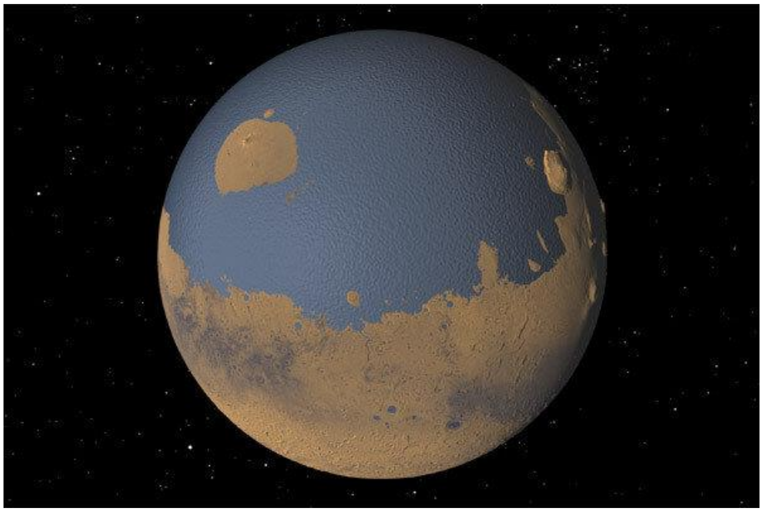
(a) Determine the value of the constant k for which f(t) is in fact a probability density function.

(b) Determine the probability that a Martian eel was *longer* than 1 meter.

(c) Determine the probability that a Martian eel was *less than* 2 meters in length.

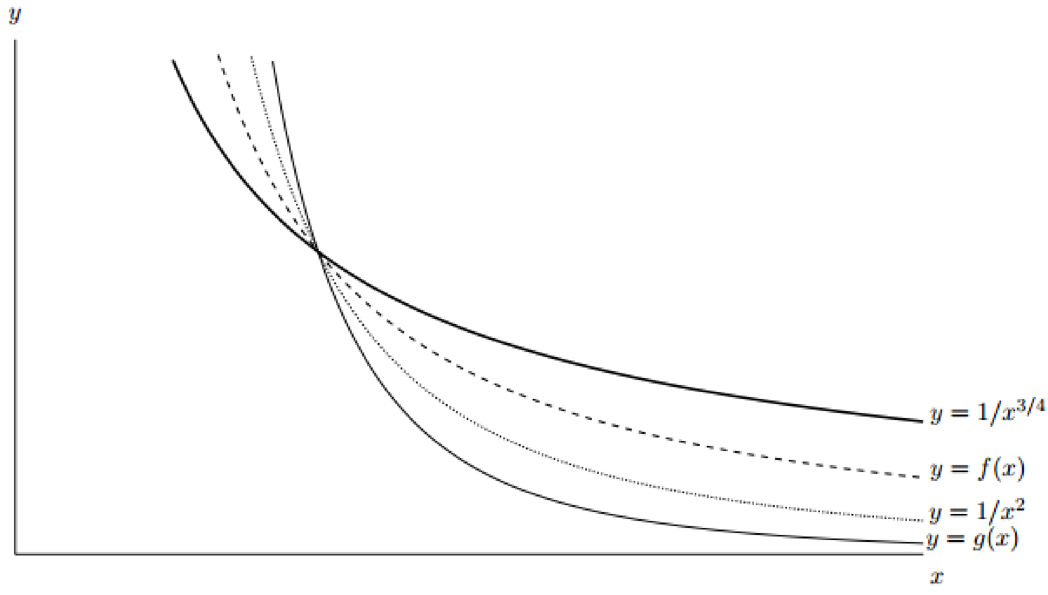
(d) Find the *expected value* of the length of a Martian eel.

(e) Find the *median* of the eel's length.



An artist rendering of what recent research suggests Mars might have looked like around 4 billion years ago, when most researchers think the planet was considerably warmer than it is today. CreditGreg Shirah/NASA

12. Consider the graph below depicting four functions for x> 0. The only point of intersection between any two of the functions is at x = 1. The functions f(x) and g(x) are both differentiable, and they each have y = 0 as a horizontal asymptote and x = 0 as a vertical asymptote.



Use the graph to determine whether the following quantities converge and diverge. If there is not enough information to determine convergence or divergence, write "Not enough information." Explain your reasoning.

$$(a)\quad \int_{1}^{\infty} f(x)\,dx$$

$$(b)\quad \int_{0+}^{1} g(x)\,dx$$

$$(c)\quad \int_{0+}^{1} g'(x)\, e^{-g(x)}\,dx$$

$$(d)\quad \int_{1}^{\infty} \sqrt{g(x)}\,dx$$

Little o, big o,

Integration by parts

Recursion

"Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?" "I don't know," said Alice. "I lost count."

- Lewis Carroll, **Through the Looking Glass**