## WRITTEN ASSIGNMENTS FOR MATH 161 HW

## HW C (due Monday, $\mathbf{2 6}^{\text {th }}$ February) (revised 2/25)

1. Albertine is studying the Sierpinski Carpet which is an example of a mathematical object called a fractal. It is constructed by removing the center one-ninth of a square of side 1 , then removing the centers of the eight smaller remaining squares, and so on. (The figure below shows the first three steps of the construction.)


At the $n^{\text {th }}$ step of the process, $8^{n-1}$ squares are removed, each with area $1 / 9$ inches. Thus, the area removed at the $n^{\text {th }}$ step is $A_{n}=\frac{8^{n-1}}{9^{n}}$. There are infinitely many steps in the process.
(a) Find the limit of the sequence $A_{1}, A_{2}, A_{3}, \ldots$.
(b) Write a mathematical expression that represents $A$, the total sum of the areas of the removed squares after infinitely many steps of the process.
(c) Exactly how much area is removed in all? Show your work
2. In two or three paragraphs, explore the concept of Golden Ratio. Discuss its relationship to the Fibonacci sequence and its importance in art, music, architecture, and science.
3. Evaluate each of the following integrals (without using Mathematica):

$$
\int \sqrt{\sqrt{-\sqrt{x \sqrt{x \sqrt{x}}}}} d
$$

$$
\int_{0}^{2018 \pi}|\sin (2018 x)| d x
$$

