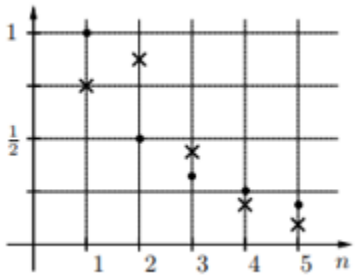
# MATH 162: HOMEWORK E

1. Albertine is trying to find all solutions to the equation. She needs your help!

$$1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \frac{(3x)^6}{6!} + \cdots = 0.$$

You must show your work clearly and give exact answers. Calculator approximations or methods will receive no credit.

2. Let $a_n$ and $b_n$ be the two sequences shown in the figure below. The sequence $a_n = \frac{1}{n}$ is shown with solid dots ● and the sequence $b_n$ is shown with crosses **X**. For $5 \leq n < \infty$, $0 < b_n < a_n$,




(a) Does the sequence $b_n$ converge, diverge, or can we not tell? Explain each answer in one or two complete sentences. If the sequence converges, indicate the value to which it converges.

(b) Does the series $\sum_{n=1}^{\infty} b_n$ converge, diverge, or can we not tell? Explain in one or two complete sentences. If the series converges, indicate the value to which it converges.

3. Suppose that we know that $\sum_{n=1}^{\infty} a_n$ converges --- but we don't know what $a_n$ is. For each of the series below, determine whether the series converges, diverges, or we cannot tell (that is, there could be one sequence $\{a_n\}$ for which the series converges and another for which the series diverges.

(a) $\sum_{n=1}^{\infty} |a_n| \sum_{n=1}^{\infty} \frac{a_n}{n^2}$

(b) $\sum_{n=1}^{\infty} (-1)^n |a_n|$

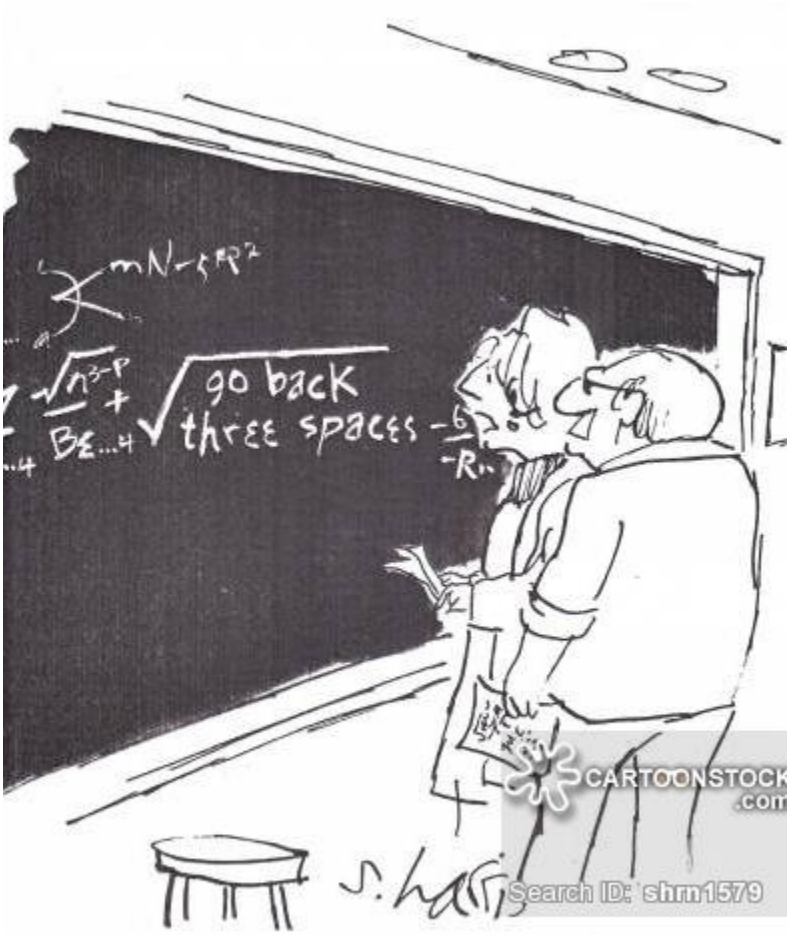
(c) $\sum_{n=1}^{\infty} \frac{a_n+1}{a_n+5}$

(d) $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$

(e) $\sum_{n=1}^{\infty} \frac{3^n a_n}{n^3}$

4. Albertine, Gilberte, and Swann are obsessed by the following problem. Please help them solve this problem.

$$\lim_{n\to\infty} I_n, \text{ where } I_1 = \int_0^1 \frac{dx}{1+\sqrt{x}}, \quad I_2 = \int_0^1 \frac{dx}{1+\frac{1}{1+\sqrt{x}}}, \quad I_3 = \int_0^1 \frac{dx}{1+\frac{1}{1+\frac{1}{1+\sqrt{x}}}}, \quad \cdots$$

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