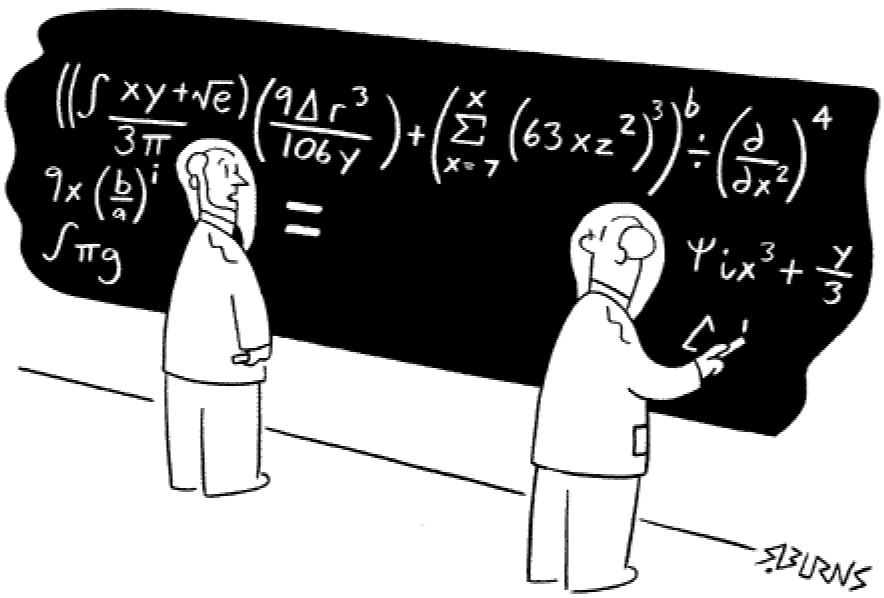
DISCUSSION: 24 JANUARY

IMPROPER INTEGRALS (REVISED)



"What's the square root of infinity again?"

1. Explain what is meant by "improper integral of the first kind" and "improper integral of the second kind." What does it mean to say that an improper integrals *converges*? *diverges*? *converges to the limit L*?

2. Discuss the comparison test for improper integrals. (Is there a difference in dealing with improper integrals of the *first kind* vs improper integrals of the *second kind*?)

3. Calculate the *exact* value of each of the following improper integrals of the first kind. (Each converges.)

$$(A)\quad \int_1^{\infty} \frac{1}{x^{\pi}}\, dx$$

$$(B)\ \int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$$

$$(C) \int_0^{\infty} te^{-t^2} dt$$

$$(D) \int_0^{\infty} \frac{v}{(1+v^2)^4}\, dv$$

4. For which values of p does each of the following converge?

$$(A) \quad \int_1^{\infty} \frac{1}{x^p}\, dx$$

$$(B) \quad \int_e^{\infty} \frac{1}{x(\ln x)^p}\, dx$$

$$(C) \quad \int_3^{\infty} \frac{1}{x(\ln x)(\ln \ln x)^p}\, dx$$

$$(D) \quad \int_0^{\infty} e^{-px}\, dx$$

4. For each of the following improper integrals of the first kind, determine convergence or divergence. In each case, carefully explain how you obtained your answer.

$$(A) \quad \int_0^{\infty} \sin^2 x\, dx$$

$$(B) \quad \int_2^{\infty} \frac{1}{x+\sin x}\, dx$$

$(C) \quad \int_{-\infty}^{\infty} \exp(-x^2)\, dx$ *Note:* Recall that exp(f(x)) means $e^{f(x)}$.

$$(D) \quad \int_0^{\infty} \frac{9+91x^5+2018\sqrt{x}}{1+x^8}\, dx$$

$(E)\quad \int_0^\infty \frac{1+e^x}{1+x^{1000}}\,dx$

$(F)\quad \int_2^\infty \frac{\cos^4 x}{x^2+x+1}\,dx$

$(G)\quad \int_0^\infty \frac{1+e^{2x}}{1+e^{3x}}\,dx$

$(H)\quad \int_0^\infty \frac{1+x+2x^2}{3+5x+9x^2+19x^3}\,dx$

$(I)\quad \int_1^\infty \frac{\ln x}{x^3}\,dx$

$(J)\quad \int_0^\infty \frac{x^2}{e^x}\,dx$

$(K)\quad \int_1^\infty \frac{1+e^{-x}}{x}\,dx$

$(L)\quad \int_e^\infty \frac{1}{\ln x}\,dx$

$(M)\quad \int_1^\infty \frac{x^2+\ln x}{(\ln x)^4+x^2+\sqrt{x}+13}\,dx$

5. For which values of p does the following improper integral converge?

$$\int_{0+}^1 \frac{1}{x^p}\,dx$$

6. For each of the following improper integrals of the *second kind*, determine converge or divergence. In each case, carefully explain how you obtained your answer.

$(A)\quad \int_{0+}^1 \frac{11+x^2}{x^3}\,dx$

$(B)\quad \int_0^{1-} \frac{1}{\sqrt{1-x^2}}\,dx$

$$(C) \quad \int_0^{\frac{\pi}{2}-} \tan x \, dx$$

$$(D) \quad \int_{0+}^{1} \ln\left(\frac{1}{x}\right) dx$$

$$(E) \quad \int_{0+}^{1} \frac{1+x+x^5}{x^9} \, dx$$

7. How do *little oh* and *big oh* help us to implement the Comparison Test for improper integrals?