Handout 1 August 25, 2015

What constitutes a definition?

For example, let p and q be integers. We say that ***p is divisible by q*** if: ?

Let n be an integer. We define n to be **even** if: ?

We define n to be **odd** if: ?

We define an integer $n \geq 2$ to be a **prime number** if:

What about the following "definitions"?

(a) Define a function *f* on the real line as follows: $f(x) = \begin{cases} 8x \text{ if } x \text{ is a positive real} \\ 9x - 13 \text{ if } x \text{ is a negative real} \end{cases}$

(b) Define a function *g* on the real line as follows: $g(x) = \begin{cases} 8/x \text{ if } x \text{ is a positive real} \\ x^3 \text{ otherwise} \end{cases}$

What constitutes a proof?

Proposition: If an integer *n* is divisible by 8, then *n* must be even.

"Proof" 1: If a number is divisible by 8, that means 8 goes into it without remainder. Since 2 goes into 8 without a remainder, that means that 2 goes into the original number without a remainder.

"Proof" 2: If a number is divisible by 8, it has to be equal to 8 times some other number. But since 8 is even, if you take any number and multiply it by 8, you get an even number, so the original number is even.

"Proof" 3: If n is divisible by 8, then there is some number d such that n = 8d. But since 8 is even, 8d is even, so n is even.

"Proof" 4: Let n be an integer that is divisible by 8. Then there exists an integer d such that n = 8d. So n = 2(4d) and 4d is an integer. Hence n = 2m where m is an integer, and so n is even.

What is a conjecture? For example: Observe that 24 = 5 + 19 = 7 + 17 = 11+ 13

8 = 3 + 5

38 = 19 + 19 = 7 + 31

Can you formulate a conjecture from the above examples?