Homework Solutions: Problem set 1 (section 1.4 of text)

**4. Prove that the sum of two odd numbers n1 and n2 is even.**

Proof: Let n1 and n2 be odd integers. Then (by definition of odd integer) there exist integers a and b such that

n1 = 2a + 1 and n2 = 2b + 1.

Now: n1 + n2  = (2a + 1 ) + (2b + 1) = 2a + 2b + 2 = 2 ( a + b + 1).

Next, note that q = a + b + 1 is an integer (since Z is closed under addition).

Thus n1 + n2  = 2q which is even (by definition of even integer).

**9. Scary Clown offers a Sad Meal containing a sandwich, a salad, a dessert, and a drink. (They are not mixed together in the box.) There are 11 types of sandwiches, 3 types of salads, and 5 different kinds of desserts. A person with low standards for food could eat a different Sad Meal every day for three years. So how many drinks are possible choices for a Sad Meal?**

Solution: Let A = set of the types of sandwiches that are available. Then |A| = 11.

Let B = set of the types of salads that are available. Then |B| = 3.

Let C = set of the types of desserts that are available. Then |C| = 5.

Let x = number of different types of drinks that must be made available to our customer with low standards.

Using the Multiplication Principle, the number of Sad Meals available is:

|A| |B| |C| x = (11)(3)(5)x

Now in three years there are at most (365 + 365 + 366) = 1096 days.

Thus we need the number of unique Sad Meals to be at least 1096:

(11)(3)(5) x ≥ 1096

From this we see that x ≥ 1096/{(11)(3)(5)} = 6.64.

Now since x must be an integer, we must choose x = 7 (or larger) to satisfy our customers’ requirements.

**12**. **Prove, or find a counterexample to: the sum of two perfect squares is even.**

Counterexample: Clearly 32 = 9 and 42 = 16 are perfect squares. But 9 + 16 = 25 = 2(12) + 1 which is odd.