Homework Solutions: Problem set 1 (section 1.4 of text)

4. Prove that the sum of two odd numbers n_1 and n_2 is even.

Proof: Let n_1 and n_2 be odd integers. Then (by definition of odd integer) there exist integers a and b such that

$$n_1 = 2a + 1 \text{ and } n_2 = 2b + 1.$$

Now: $n_1 + n_2 = (2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1)$.

Next, note that $q = a + b + 1$ is an integer (since $\mathbb{Z}$ is closed under addition).

Thus $n_1 + n_2 = 2q$ which is even (by definition of even integer).

9. Scary Clown offers a Sad Meal containing a sandwich, a salad, a dessert, and a drink. (They are not mixed together in the box.) There are 11 types of sandwiches, 3 types of salads, and 5 different kinds of desserts. A person with low standards for food could eat a different Sad Meal every day for three years. So how many drinks are possible choices for a Sad Meal?

Solution: Let A = set of the types of sandwiches that are available. Then $|A| = 11$.

Let B = set of the types of salads that are available. Then $|B| = 3$.

Let C = set of the types of desserts that are available. Then $|C| = 5$.

Let x = number of different types of drinks that must be made available to our customer with low standards.

Using the Multiplication Principle, the number of Sad Meals available is:

$$|A|\,|B|\,|C|\,x = (11)(3)(5)x$$

Now in three years there are at most (365 + 365 + 366) = 1096 days.

Thus we need the number of unique Sad Meals to be at least 1096:

$$(11)(3)(5)\,x \geq 1096$$

From this we see that $x \geq 1096/\{(11)(3)(5)\} = 6.64$.

Now since x must be an integer, we must choose x = 7 (or larger) to satisfy our customers' requirements.

12. **Prove, or find a counterexample to: the sum of two perfect squares is even.**

Counterexample: Clearly $3^2 = 9$ and $4^2 = 16$ are perfect squares. But $9 + 16 = 25 = 2(12) + 1$ which is odd.