Solutions: Homework 2 (section 1.7 of text)

11.   Every US coin is stamped with the year in which it was minted. How many coins do you need to have in your pocket to be assured that at least two of them must have the same last digit? How many do you need to be assured that at least two have the same first digit?

Solution: Here we use the Pigeon-Hole Principle. If pigeon holes represent the (ten) digits from 0 through 9, and the pigeons represent 11 coins, then there must be two pigeons in one of the ten pigeon holes. Any fewer than 11 coins and this result is no longer the case.   Thus 11 is the minimum number of coins needed.

In the second part of this problem, we need only two pigeon-holes representing the integers 1 and 2. Thus 3 pigeons will guarantee that two will end up in the same pigeon hole.

In other words, we would need 3 coins to be certain that two of them have the same first digit.

17.   At Chicago O'Hare International Airport, there is an average of 1042 flights per day. Prove that at least two of these flights must take off with 90 seconds of each other.

Proof: Partition the day into 90 second intervals.   We will have 24(60)(60) / 90 = 960 intervals in one day. Let each such interval be a pigeon-hole.   Let the pigeons represent the 1042 planes that take off during this day,   Since 1042 > 960, the Pigeon-Hole Principle guarantees that at least 2 pigeons will fly into the same pigeon hole.

In other words, there will be two planes that takeoff within 90 seconds of each other.

23.   **Prove that if *n* is any integer, then $3n^3 + n + 5$ is odd.**

Proof: We will split this proof into two cases.

Case I: n is odd.   Then $\exists d \in Z$ such that n = 2d + 1.

Now: $3n^3 + n + 5 = 3(2d + 1)^3 + (2d + 1) + 5 =$

$3(8d^3 + 12d^2 + 6d + 1) + (2d + 1) + 5 =$

$24 d^3 + 36 d^2 + 20d + 9 =$

$2(12d^3 + 18\ d^2 + 10d\ + 4) + 1$ which is odd since $12d^3 + 18d^2 + 8d + 8 \in$ Z

Case II: n is even.   Then $\exists d \in Z$ such that n = 2d.

Now $3(2d)^3 + (2d) + 5$ =

$24\ d^3 + 2d + 5 =$

$24\ d^3 + 2d + 4 + 1 =$

$2(12\ d^3 + d + 2) + 1$ which is odd since $12\ d^3 + d + 2 \in Z$

Thus $\forall\ n\ \in Z$ $3n^3 + n + 5$ is odd.