**Solutions: Homework 3 (section 2.2 of text)**

**11. Give a counterexample to the statement** $\left|A∪B\right|=\left|A\right|+\left|B\right|.$

Solution: Let A = B = {0, 1}. Then $A∪B=\left\{0, 1\right\}.$ Now |A| = 2, |B| = 2, and $\left|A∪B\right|=2.$

Clearly $2=\left|A∪B\right|\ne \left|A\right|+\left|B\right|=4.$

**14. Decide whether or not it is true that** $(A×B)∪\left(C×D\right)=(A∪C)×(B∪D)$

Solution: This is False. (Almost any choice of A, B, C, D will serve you well.)

Let A = C = Ø, the empty set and B = D = {1}.

Then $A×B=∅, C×D= ∅ , A∪C=∅, and B∪D=\left\{1\right\}.$

Thus $\left(A×B\right)∪\left(C×D\right)=∅, yet \left(A∪C\right)∪\left(B∪D\right)=\left\{1\right\}.$

Note: Perhaps an easier way to view this is by letting A = B = [0, 1] and C = D = [1, 2], two intervals on the *real line*. (We know this is not a discrete set, but it works nonetheless.)

Then $A×B$ is a square in the plane; so is$ C×D$. But the union of these two squares is not a square, whereas the right-hand side of our set equality above is a square of side length 4.

(Draw a picture!)

**18**. **Write this in English:** $∀k\in 3Z, ∃S⊆N, \left|S\right|=k.$ **(Is it true?) What is the negation of this statement? (Is the negation true?)**

Solution: For every integer, *k*, divisible by 3, there exists a set of *k* counting numbers.

 *This is not a true statement: just let k = -3.*

The negation says: There exists an integer, *k*, divisible by 3, such that no set of *k* counting numbers exists.

 As a logical sentence, this negation is: $∃k\in Z, ∀ S⊆N, \left|S\right|\ne k. $

Clearly this is a true statement since it is the negation of a false statement.