# Solutions: Homework 3 (section 2.2 of text)

**11. Give a counterexample to the statement $|\boldsymbol{A} \cup \boldsymbol{B}| = |\boldsymbol{A}| + |\boldsymbol{B}|$.**

Solution: Let A = B = {0, 1}. Then $A \cup B = \{0, 1\}$. Now |A| = 2, |B| = 2, and $|A \cup B| = 2$.
Clearly $2 = |A \cup B| \neq |A| + |B| = 4$.

**14. Decide whether or not it is true that $(\boldsymbol{A} \times \boldsymbol{B}) \cup (\boldsymbol{C} \times \boldsymbol{D}) = (\boldsymbol{A} \cup \boldsymbol{C}) \times (\boldsymbol{B} \cup \boldsymbol{D})$**

Solution: This is False. (Almost any choice of A, B, C, D will serve you well.)

Let A = C = ∅, the empty set and B = D = {1}.

Then $A \times B = \emptyset,\ C \times D = \ \emptyset\ ,\ A \cup C = \emptyset, and\ B \cup D = \{1\}$.

Thus $(A \times B) \cup (C \times D) = \emptyset,\ yet\ (A \cup C) \cup (B \cup D) = \{1\}$.

Note: Perhaps an easier way to view this is by letting A = B = [0, 1] and C = D = [1, 2], two intervals on the *real line*. (We know this is not a discrete set, but it works nonetheless.)

Then $A \times B$ is a square in the plane; so is $C \times D$. But the union of these two squares is not a square, whereas the right-hand side of our set equality above is a square of side length 4. (Draw a picture!)

**18. Write this in English: $\forall \boldsymbol{k} \in 3\mathbb{Z}, \exists \boldsymbol{S} \subseteq \mathbb{N}, |\boldsymbol{S}| = \boldsymbol{k}$. (Is it true?) What is the negation of this statement? (Is the negation true?)**

Solution: For every integer, *k*, divisible by 3, there exists a set of *k* counting numbers.

*This is not a true statement: just let k = -3.*

The negation says: There exists an integer, *k*, divisible by 3, such that no set of *k* counting numbers exists.

As a logical sentence, this negation is: $\exists k \in \mathbb{Z}, \forall\ S \subseteq \mathbb{N}, |S| \neq k$.

Clearly this is a true statement since it is the negation of a false statement.