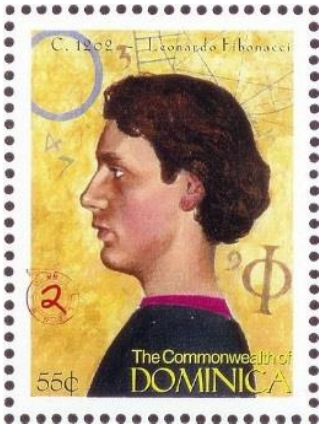
PROBLEM SET 11: RECURRENCE RELATIONS

Let F_n be the n[th] Fibonacci number. (Let us assume that $F_1 = F_2 = 1$.)



1. Using induction prove that, for all n∈N, $1 + \sum_{k=1}^{n} F_k = F_{n+2}$

2. Using induction prove that, for all n∈N, $F_{n-1}F_{n+1} = (F_n)^2 + (-1)^n$

3. Find a formula for the sum of the first n odd-index Fibonacci numbers (F_1, F_3, F_5, ...). Prove your conjecture.

4. Find a formula for the sum of the first n even-index Fibonacci numbers (F_2, F_4, ...). Prove your conjecture.

5. Prove that $F_{n+3} = 2F_{n+1} + F_2F_n$

6. Find a recurrence formula that defines the sequence 2, 5, 8, 11, 14, ...

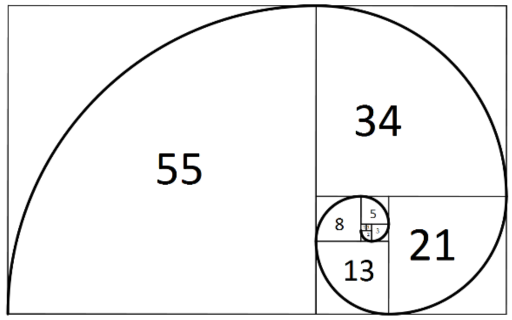
7. Find a closed form expression for each of the following sequences:

   (a) 1, -3, 9, -27, 81, ...

   (b) -6, -1, 4, 9, 14, ...

   (c) 1, 8, 27, 64, 125, ...

8. Find a closed form expression for the sequence: $a_0 = 8$, $a_n = a_{n-1} - 4$



Fibonacci spiral

Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.

- Leohard Euler (1707-1783)