# Problem set 15: Cardinality revisited *(revised)*

& the Cantor-Schroeder-Bernstein Theorem



1. Show that each of the following sets has cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png (read Aleph-Zero).
2. N {0, -1, -2, -3}
3. Z, the set of integers
4. A B where A and B are disjoint and each has cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.
5. Let Aj (j = 1, 2, ..., 13) be pairwise disjoint sets each of cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.

Prove that has cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.

1. Let Aj (j = 1, 2, 3, ...) be a sequence of pairwise disjoint sets each of cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.

Prove that has cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.

1. A B where each of A and B has cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png.
2. Prove that the set of rational numbers Q is of cardinality https://upload.wikimedia.org/wikipedia/commons/thumb/e/e0/Aleph0.svg/2000px-Aleph0.svg.png
3. What is Cantor’s ***Infinite Hotel****? Read Vilenkin’s* [**In Search of Infinity**](https://math.dartmouth.edu/~matc/Readers/HowManyAngels/SearchInfinity.html)**.**
4. Prove that the set of real numbers is *uncountable*. (Why is this called Cantor’s *diagonal* *argument*?)
5. *True or False?* Justify each answer!
6. The set of all complex numbers, a + bi, where a and b are integers, is countable.
7. The set of all complex numbers, a + bi, where a and b are rational numbers, is countable.
8. The set of all numbers of the form is uncountable.
9. If A is countable and A B then B is countable.
10. If A is uncountable and A B then B is uncountable.
11. If A is countable and B is uncountable then A B is uncountable.
12. The set of all irrational numbers is uncountable.
13. If there exists an injection F: AB, then the cardinality of A cannot be less than the cardinality of B.
14. If there exists a surjection G: AB, then the cardinality of A cannot be less than the cardinality of B.
15. State the **Cantor-Schroeder-Bernstein Theorem**. Study its proof given below.

(from: [Art of problem solving](http://www.artofproblemsolving.com/))

