Math 201   Problem sheet 2   27 August 2015   Name:________________   Group:_____

PIGEON-HOLE PRINCIPLE (REVISED)

PART I: State the basic pigeon-hole principle.

State the strong pigeon-hole principle (*n* pigeon holes, and at least *kn+1* pigeons).

Solve each of the following problems by using the pigeon-hole principle.

(a) A bag contains balls of five colors: blue, purple, black, green and red. What is the *smallest* number of balls that must be drawn from the bag (without looking) so that among the drawn balls there are at least two of the same color? *(Who are the pigeons and what are the pigeon holes?)*

(b) There are 800,000,000 pine trees in Birnam Wood. Each pine tree has no more than 600,000 needles. Show that at least two trees in the forest have the same number of needles.

(c) Fifteen students in French 103 were given a dictation quiz. Albertine made 13 errors. Each of the other students made fewer errors. Prove that *at least two* students made the same number of errors. *(Who are the pigeons and what are the pigeon holes?)*

(d) There are 30 students in Spanish 103. On a dictation quiz, Gilberte made 13 errors and all the other students made fewer errors. Prove that *at least three* students made the same number of errors.

(e) Given 12 non-negative integers, show that two of them can be chosen whose difference is divisible by 11. (*Hint:* Let the pigeons represent the twelve integers. If a non-negative number is divided by 11, how many possible remainders may occur?

(f) There are 50 people in a room. Some of them are acquainted with each other, some not. (Assume that "acquainted with" is a symmetric but not a reflexive relation.)

Note: **reflexive** would mean that Albertine knows Albertine; **symmetric** means that if Albertine knows Boris then Boris knows Albertine.

Prove that there are two persons in the room who have an equal number of acquaintances. (*Hint:* Let the pigeons be the 50 people. Consider two cases: Either everyone is acquainted with at least one other person, or else at least one person has no acquaintances.)

(g) There are five points inside an equilateral triangle of side length 2. Show that at least two of the points are within 1 unit distance from each other.

Hint: Draw that triangle connecting the midpoints of the three sides of the given triangle.

PART II: The **power set** of a set A is defined to be the set of all subsets of A. The power set of A is denoted by ₱(A). Recall that if A is a finite set then |A|, the number of members of A, is referred to as the **cardinality of A**.

(a) Let A = {0, 1, 2}, Find ₱(A) by listing all the subsets of A

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(b) Let B = {Jack, Queen, King, Ace}. Find ₱(B)

(c) If |C| = n, what is the cardinality of the power set of C?

(d) If D is the empty set (denoted by φ), what is the cardinality of Ᵽ(D)?

(e) If E = {φ}, list the elements of the power set of E.

(f) If F = {1, {1}, {3, 4}} list all the elements of the power set of F.

Johann Peter Gustav Lejeune Dirichlet first stated the pigeon-hole principle (also known as *Dirichlet's box principle*) in 1834.

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