Math 201 Problem set 5 8 September 2015

I Solve the following exercises from our text:

Check Yourself

The sum principle reveals that what seem like two problems are in truth seven.

1. Here are some gipos that have domain $\mathbb{N}$. For each gipo, determine whether it is a function, whether the target space is also $\mathbb{N}$, and whether it is one-to-one.

(a) $f(n) = \frac{n}{3} + 1.$

(b) $f(n) = n.$

(c) $f(n) = n - 1.$

(d) $f(n) = n^2 - 1.$

2. Here are some functions that have domain $\mathbb{Z}$ and target space $\mathbb{W}$. For each function, determine whether it is one-to-one or onto.

(a) $f(k) = 0.$

(b) $f(k) = \left| \lfloor \frac{k}{2} \rfloor \right|.$ (The notation $\lfloor x \rfloor$ is known as the *floor* function, as it returns the integer equal to or just less than the input. Thus, $\lfloor \frac{k}{2} \rfloor$ returns $\frac{k}{2}$ if k is even and $\frac{k-1}{2}$ if k is odd.) (Oh, and there is a matching *ceiling* function, which returns the integer equal to or just greater than the input.)

(c) $f(k) = k^2 + 2.$

For those functions that are not onto, what is the range? Are any of the functions bijections?

II In the following, let **N**, **Z**, **Q**, **R** denote the set of natural numbers, the set of integers, the set of rational numbers, and the set of real numbers, respectively. For each function "candidate" below, first determine if it is well-defined. If so, then determine if it enjoys any of the properties of being injective, surjective, bijective.

(a) F: $\mathbf{N} \to \mathbf{N}$ given by F(n) = n + 1

(b) G: $\mathbf{N} \to \mathbf{N}$ given by G(n) = n – 1

(c) H: $\mathbf{Z} \to \mathbf{N}$ given by H(m) = |m|

(d) f: $\mathbf{Z} \rightarrow \mathbf{N}$ given by $f(m) = |m| + 1$

(e) α: $\mathbf{Q} \rightarrow \mathbf{Z}$ given by $\alpha(a/b) = a + b$

(f) β: $\mathbf{Q} \rightarrow \mathbf{Z}$ given by $\beta(x) = ab$ where $x = a/b$ (where *a* and *b* are nonnegative) or $-a/b$ (where *a* and *b* are nonnegative) and $\gcd(a,b)=1$

(g) F: $\mathbf{N} \rightarrow \mathbf{N}$ given by $F(n) = n^2$

(h) G: $\mathbf{R} \rightarrow \mathbf{R}$ given by $G(x) = (x - 1)(x - 2)(x - 3)$

(i) H: $\mathbf{Z} \rightarrow \mathbf{Z}$ given by $H(m) = z + 11$

(j) id: $\mathbf{X} \rightarrow \mathbf{X}$ given by $id(m) = m$

(k) p: $\mathbf{N} \rightarrow \mathbf{Q}$ given by $p(j) = 1/j$

(l) F: $\mathbf{N} \rightarrow \mathbf{Q}$ given by $F(n) = n/13$

(m) z: $\mathbf{N} \rightarrow \mathbf{N}$ given by z(m) = sum of the digits in the decimal representation of *m*.

III Let A and B be finite sets and let *f* be a function f: $A \rightarrow B$. Prove that if $|A| > |B|$ then *f* cannot be injective.