Math 201: problem set VI

Isomorphism

1. Let X be a set endowed with operation ★ and let Y be a set endowed with operation . (It is assumed, of course, that each set is closed under its own operation).

(Sometimes X and Y will have multiple operations.)

An isomorphism is a bijection : X → Y that preserves operations.

More precisely: For all x1 and x2 X, ( x1★ x2) = (x1) (x2).

For example, let X be the space of 21 (column) vectors with real entries, endowed with the usual matrix addition. Let Y be the space of 1 2 (row) vectors, also endowed with the usual matrix addition.

Define $Φ\left(\left[\begin{matrix}a\\b\end{matrix}\right]\right)=\left[\begin{matrix}a&b\end{matrix}\right]$

Prove that $Φ$ is an isomorphism.

1. Consider the following function : → defined by the rule, for all a, $Φ$ $\left(a\right)=a+1$.

Assume that operation on Z is addition in both the domain and target.

1. Is well-defined?
2. Is surjective?
3. Is injective?
4. Is bijective?
5. Is an isomorphism?

 *Investigate each of the following functions by going through the list of five questions in #2.*

1. Let X be the space of all polynomials with real coefficents.

Let X and Y have one operation: polynomial addition.

Define F: X → Y by the following rule: for all p X, F(p) = F’

1. Are the following two graphs isomorphic?



1. Let X be the set of polynomials of degree 2 or smaller with real coefficients, endowed with the usual addition of polynomials. Let Y be the set of 31 column vectors, endowed with the usual matrix addition. Define G: X → Y as follows: For all a, b, cR G(a + bx + cx2) = $\left[\begin{matrix}a\\b\\c\end{matrix}\right]$.
2. Let X and Y each be the set of complex numbers endowed with the usual addition and multiplication. Note, here each set has two operations.

Define H: X → Y as follows: For all zX H(z) = conjugate of z.

1. Let X be the set of real numbers endowed with the usual addition. Let Y be the set of all non-negative real numbers endowed with the usual multiplication.

Define T: X → Y as follows: For all sX T(s) = 2s.

1. Are the following two graphs isomorphic?



11. Are the following two graphs isomorphic?



*The origins of graph theory are humble, even frivolous.*- N. Biggs, E. K. Lloyd, and R. J. Wilson (*Graph Theory: 1736-1937)*