# Problem set 9: Review questions

1. *Critique* the proof below: Proof is correct as is? Error in algebra or arithmetic? Stylistic errors? Logical errors?

**Proposition**: n ∈ N, 12 + 22 + 32 + ... + n2 = n(n+1)(2n+1)/6

**Proof:** For any n, let Sn be the statement 12 + 22 + 32 + ... + n2 = n(n+1)(2n+1)/6.

**Base Case:** S1 is true, since 12 = 1(1 + 1)(2 + 1)/6

**Inductive Step:**

Assume that Sk is true for k N.

Then 12 + 22 + 32 + ... + (k+1)2 = (k+1)(k+1+1))(2(k+1)+1)/6

k(k+1)(2k+1)/6 + (k+1)2 = (k+1)(k+1+1))(2(k+1)+1)/6

Factoring: (1/6)(k+1){k(2k+1) + 6(k+1) = (k+1)(k+2)(2k+3)/6

(1/6) (k+1)(2k2 + 7k + 6) = (k+1)(k+2)(2k+3)/6

Factoring the left-hand side:

(k+1)(k+2)(2k+3)/6 = (k+1)(k+2)(2k+3)/6

This is a true statement!

Thus we have shown that Sk is always true.

2. *Evaluate the proof of the following statement:*

**Proposition**: For all n ∈ N, 4|(32n + 7).

Proof: For n ∈ N let Sn be the statement 4|(32n + 7).

**Base Case:** S1 is true, since 32 + 7 = 16 is divisible by 4.

**Inductive Step:**

For a given k ≥ 1, assume that Sk is true, so that 4|(32k + 7). Then 32k + 7 = 4L for some L∈ Z.

Now, 32(k+1) + 7 = 9(32k ) + 7 = 8(32k ) + 32k + 7 = 8(32k ) + 4L = 4(2(32k ) + L).

So 4|(32(k+1) + 7), and we see that Sk+1 is true.

Hence S~~k~~ implies Sk+1.

Therefore, by the principle of mathematical induction, Sn is true for all n ∈ N.

🡪 Which of the following statements is correct?

a) The proposition is false but the proof is correct.

b) The proof contains arithmetic mistakes which make it incorrect.

c) The proof incorrectly assumes what it is trying to prove.

d) The proof is a correct proof of the stated result.

e) None of the above.

**3.** (a) Show that there is no integer x satisfying the equation 2x + 1 = 5x – 4

(Hint: evaluate mod 3 and see what happens)

(b) Show that there is no integer x satisfying the equation 18x2 + 39x – 7 = 0

(c) Show that the system of equations

11x – 5y = 7

9x + 10y = -3

has no integer solution. (Hint: reduce mod 5)

(d) Show that the system of equations

24*x* – 5*y* = 10,
11*x* – 9*y* = 13.

has no integer solutions.

4. Let X, Y, Z be non-empty sets. Let f: X → Y be surjective and let

g: Y → Z be surjective. Must it follow that the composition of the two functions

g∘f : X →Z be surjective? *Give proof or counterexample.*

5. Is the converse to problem # 2 true? *Give proof or counterexample.*

(Begin by stating the converse!)

6. (a) Is 4100 divisible by 3? (use modular arithmetic)

(b) What is the last digit in the expansion of 4100 ?

7. (a) Using *mathematical induction*, prove that 10n+1 + 4(10n ) + 4 is divisible by 9, for all positive integers n.

(b) Prove the result of part (a) by using *modular arithmetic*!

Which is easier?

8. Let A = {1, 2, 3, 4, 5}, and let

*R* = {(1, 1), (1, 3), (1, 4), (2, 2), (2, 5), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4), (5, 2),

(5, 5)} define an equivalence relation on A. (You may wish to check this!)

Which of the following is an *equivalence class*?

1. {1, 2, 3} b) {2, 3, 5} c) {1, 3, 4} d) {1, 2} e) {1, 2, 3, 4, 5}

 *I tell them if they will occupy themselves with the study of mathematics, they will find in it the best remedy against the lusts of the flesh.*

- Thomas Mann, **The Magic Mountain**

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