MATH 201 Solutions: QUIZ I

**I**  [2 pts each] **TRUE** or **FALSE** (You *need not justify your answers*.)

1. Let A, B and C be sets. Then A (B C) = (A B) (A C).

***False***

*Counterexample: Let A = {1}, B = {-2, 1, 3}, C = {0, 3}.*

*Then LHS = {1} and RHS = {0, 1, 3}*

1. Let P and Q be propositions. Then P (P Q) is a tautology.

***True***

*The truth table has only values of True for this implication.*

1. Let A and B be finite sets. Then there exists an injection F: A → B only if

|A| ≤ |B|.

***True****, using the pigeon-hole principle.*

1. If A is a set of cardinality 10, then the cardinality of the power set of A, P(A), is 102.

***False****: the correct cardinality is 210.*

1. Let A and B be finite sets. Then |A B| ≤ |A| + |B|.

***True***

*Since |A B| = |A| + |B| – |A B| ≤ |A| + |B|.*

*Or, consider a Venn diagram.*

1. Let P and Q be propositions. The contrapositive of P Q is (P) (Q).

***False***

*The contrapositive asserts that (Q) (P).*

(g)Let P and Q be propositions. ThenP Q is logically equivalent to (P) Q.

***False***

*Consider the truth tables.*

1. Let A and B be finite sets. Then |A B| = |A| |B|.

***True***

*This is one version of the multiplication principle.*

**II**  [5 pts each] Give a *clear and precise definition* of each of the following terms.

1. A *proposition* *is a minor theorem.*
2. A function F: X → Y is said to be *surjective* *if*

 *y Y x X such that F(x) = y.*

1. Let A and B be sets. Then the *union* of A and B is

*the set that contains every element that is in at least one of the two sets, A and B.*

1. The *Pigeon-Hole Principle* asserts that if

*n pigeons reside in r pigeon holes, and n > r, then at least one pigeon hole has at least 2 pigeons.*

1. Let P and Q be statements. *DeMorgan’s law* asserts that

***Set theory version:***

*if A and B are sets in a universe X, then X \ (A B) = (X \ A) (X \ B)*

*or*

*X \ (A B) = (X \ A) (X \ B)*

***Logic version:*** *Let p and q be statements.*

*Then ( p) ( q) is logically equivalent to (p q)*

*or*

*( p) ( q) is logically equivalent to (p q)*

1. Let A and B be sets. A is said to be a *proper subset* of B if

*every member of A is a member of B and A B.*

**III** [7 pts each]

(a) Let P be the proposition “Stephen Colbert is loved by all”,

Q be the proposition “Interstellar is a prophetic film”, and

R be the proposition “time is out of joint”

Express as a *sentence in English* the following logical proposition. Make certain that your sentence is *clearly written* as well as grammatically correct.

P (Q R)

*If Stephen Colbert is loved by all, then Interstellar is a prophetic film and time is not out of joint.*

1. Referring to the three propositions given in question (a), express the following sentence as a logical proposition:

*If Interstellar is not a prophetic film or time is out of joint, then not everyone loves Stephen Colbert.*

*(( Q) R) ( P)*

1. Write the *converse* of the following sentence:

 If a student earns a grade of at least B in Math 201 then she is a math major and doesn’t own an Apple Watch.

*If a student is a math major and does not own an Apple Watch, then she earns a grade of at least B in Math 201.*

1. Write the *contrapositive* of the following sentence (as an English sentence):

If George Thomas is alive then dinosaurs once roamed Paris.

*If dinosaurs never roamed Paris, then George Thomas is not alive.*

1. Let A, B, and C be sets of real numbers. *Negate* the following proposition:

 x A y B z C xyz >13

*The negation is: x A y B z C xyz 13*

*Logic is invincible, because in order to combat logic it is necessary to use logic.*

- Pierre Boutroux

*Logic doesn't apply to the real world.*

-- Marvin Lee Minsky

*Pure mathematics is, in its way, the poetry of logical ideas.*

- Albert Einstein