MATH 201 SOLUTIONS: QUIZ I

I [2 pts each] **TRUE** or **FALSE** (You *need not justify your answers*.)

(a) Let A, B and C be sets. Then $A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$.

False
Counterexample: Let A = {1}, B = {-2, 1, 3}, C = {0, 3}.
Then LHS = {1} and RHS = {0, 1, 3}

(b) Let P and Q be propositions. Then $P \Rightarrow (P \vee Q)$ is a tautology.

True
The truth table has only values of True for this implication.

(c) Let A and B be finite sets. Then there exists an injection $F: A \rightarrow B$ only if $|A| \leq |B|$.

True*, using the pigeon-hole principle.*

(d) If A is a set of cardinality 10, then the cardinality of the power set of A, $\mathcal{P}(A)$, is 10^2.

False*: the correct cardinality is* 2^{10}.

(e) Let A and B be finite sets. Then $|A \cup B| \leq |A| + |B|$.

True
Since $|A \cup B| = |A| + |B| - |A \cap B| \leq |A| + |B|$.
Or, consider a Venn diagram.

(f) Let P and Q be propositions. The contrapositive of $P \Rightarrow Q$ is $(\neg P) \Rightarrow (\neg Q)$.

False
The contrapositive asserts that $(\neg Q) \Rightarrow (\neg P)$.

(g) Let P and Q be propositions. Then $P \Rightarrow Q$ is logically equivalent to $(\neg P) \wedge Q$.

False
Consider the truth tables.

(g) Let A and B be finite sets. Then $|A \times B| = |A|\,|B|$.

True
This is one version of the multiplication principle.

II [5 pts each] Give a *clear and precise definition* of each of the following terms.

(a) A *proposition is a minor theorem.*

(b) A function $F: X \rightarrow Y$ is said to be *surjective if*

$\forall y \in Y\ \exists x \in X$ *such that* $F(x) = y$.

(c) Let A and B be sets. Then the *union* of A and B is

the set that contains every element that is in at least one of the two sets, A and B.

(d) The *Pigeon-Hole Principle* asserts that if

n pigeons reside in r pigeon holes, and $n > r$*, then at least one pigeon hole has at least 2 pigeons.*

(e) Let P and Q be statements. *DeMorgan's law* asserts that

Set theory version:
if A and B are sets in a universe X, then $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$

or

$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$

Logic version: *Let p and q be statements.*
Then $(\neg p) \vee (\neg q)$ *is logically equivalent to* $\neg (p \wedge q)$

or

$(\neg p) \wedge (\neg q)$ *is logically equivalent to* $\neg (p \vee q)$

(f) Let A and B be sets. A is said to be a *proper subset* of B if

every member of A is a member of B and $A \neq B$.

III [7 pts each]

(a) Let P be the proposition "Stephen Colbert is loved by all",
Q be the proposition "Interstellar is a prophetic film", and
R be the proposition "time is out of joint"

Express as a *sentence in English* the following logical proposition. Make certain that your sentence is *clearly written* as well as grammatically correct.

$P \Rightarrow (Q \wedge \neg R)$

If Stephen Colbert is loved by all, then Interstellar is a prophetic film and time is not out of joint.

(b) Referring to the three propositions given in question (a), express the following sentence as a logical proposition:

If Interstellar is not a prophetic film or time is out of joint, then not everyone loves Stephen Colbert.

$((\neg Q) \vee R) \Rightarrow (\neg P)$

(c) Write the *converse* of the following sentence:

If a student earns a grade of at least B in Math 201 then she is a math major and doesn't own an Apple Watch.

If a student is a math major and does not own an Apple Watch, then she earns a grade of at least B in Math 201.

(d) Write the *contrapositive* of the following sentence (as an English sentence):

If George Thomas is alive then dinosaurs once roamed Paris.

If dinosaurs never roamed Paris, then George Thomas is not alive.

(e) Let A, B, and C be sets of real numbers. *Negate* the following proposition:

$\exists\, x \in A\ \forall\, y \in B\ \exists\, z \in C \quad xyz > 13$

The negation is: $\forall x \in A\ \exists y \in B\ \forall z \in C \quad xyz \leq 13$

Logic is invincible, because in order to combat logic it is necessary to use logic.
- Pierre Boutroux

Logic doesn't apply to the real world.
-- Marvin Lee Minsky

Pure mathematics is, in its way, the poetry of logical ideas.
- Albert Einstein