**MATH 201 TEST 2-A (in class) 29 October 2015**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

MC900436145[1]

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

MC900436147[1]

*As Gregor Samsa awoke one morning from uneasy dreams he found himself transformed in his bed into a gigantic insect.*

* Franz Kafka, **The Metamorphosis**

1. The Lucas sequence, L(n), is defined recursively as follows:

L(1) = 2; L(2) = 1; L(n+1) = L(n) + L(n-1) for n 2.

*[5 pts]* List the first 10 Lucas numbers, i.e. L(1), L(2), ...., L(10)

***Extra Credit:*** Prove, using “strong induction” that L(n+2) = F(n+2) + F(n) for n 1.

Recall the principle of “strong induction”:

Let H(n) be a sequence of statements for n1

Base case: show H(1) is true

Induction step: For given n1, if H(1), H(2), ..., H(n) are true, then H(n+1) is true.

*2. [5 pts]* Consider a standard deck of 52 cards. In how many ways can one be dealt a 7-card hand (without regard to order) that contains no Queens and no Spades?

1. *[5 pts]* How many non-negative integer solutions are there to the equation:

x1 + x2 + x3 + x4 + x5 = 99?

(b) *[5 pts]* Same question as (a), but now assume that the solution must consist of *positive* integers.

***Extra credit:*** Same question as (a) except *at least one of the components* of a solution (x1, x2, x3, x4, x5)

must be 0. For example, 97 + 1 + 0 + 0 + 1 = 99 is one such solution.

4.(a) *[5 pts]* Give the *combinatorial* definition of  --- *not* the means of calculating this number or any other interpretation using factorials.

(b) *[5 pts]* Explain briefly, *using only a combinatorial interpretation*, why

 (Use complete sentences.)

(c) *[5 pts]* Explain briefly, *using only a combinatorial interpretation*, why

 (Use complete sentences.)

5**.** (a)*[5 pts]* State the *binomial theorem*.

(b)*[5 pts]* Using the binomial theorem, what is the coefficient (in terms of combinations) of x1789y226 in the expansion of (x + y)2015. (Express your answer in terms of combinations.)

(c) *[5 pts]* Compute the value. (Hint: Consider applying the binomial theorem.)

6. *[5 pts]* In the land of Oz it has been decreed that every password must be exactly 11 characters in length, contain *exactly one* capital letter (A, B, C..., Z), *exactly one* lower case letter (a, b, c, ..., z), and *exactly one* of three special characters (#, $, @). The remaining characters must be digits (0, 1, 2, .., 9).

*How many* such passwords exist?

*Extra credit:* Albertine, an Ozian, wants her password to contain digits only in strictly increasing order. So, for example, 01A23a11$87 would not be acceptable, but 02s34J5789@ is acceptable.

How many passwords are acceptable to Albertine?

7. *[5 pts]* How many ways are there to seat 14 people, consisting of 7 couples, in a row of seats (14 seats wide) if all couples demand adjacent seats?

8. *[5 pts]* How many numbers between 1 and 200 (inclusive) are divisible by 2, 3 or 5. (Do *not* solve this by “brute force.”)

9. *[5 pts]* We are given 46 balls: $ 10 $ black, 11 white, 12 red and 13 blue. In how many ways can you choose 9 balls such that exactly 3 are red?

10. *[5 pts] Find a closed form expression for the sequence:*

1, 3, 7, 13, 21, 31, 43, ...

11. *(Actuarial exam problem)*

*[5 pts]* The Legrandin insurance company has 10,000 policyholders. Each policyholder is classified as (i) young or old; (ii) male or female; and (iii) married or single. Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company’s policyholders are young, female, and single?

(A) 280 (B) 423 (C) 486 (D) 880 (E) 896

# Math 201 TEST 2-B Take-home 29 October 2015

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Instructions:** This take-home test is due on ***Thursday, November 5th, at 5 pm***. You may speak with other students about these problems, consult other texts, or the web as long as you acknowledge your sources when you turn in the test.

*Answer any 6 of the 7 problems. (You will be given extra credit for answering all seven problems.)*

*"Mine is a long and sad tale," said the Mouse, turning to Alice and sighing. "It is a long tail, certainly," said Alice, looking down with wonder at the Mouse's tail; “but why do you call it sad?"*

# - Lewis Carroll

*MC900183694[1]*Instructions: *Answer any 6 of the 7 problems. (You will be given extra credit for answering all seven problems.)*  *To receive credit, you must show your work!*

1.

MC900436145[1]

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*A first fact should surprise us, or rather would surprise us if we were not used to it. How does it happen there are people who do not understand mathematics? If mathematics invokes only the rules of logic, such as are accepted by all normal minds ... how does it come about that so many persons are here refractory?*

- Henri Poincaré, quoted in **The World of Mathematics**, by J. R. Newman.

1. There are five courses at Albertine’s new school. Students take the classes as follows: 243 take algebra; 323 take language arts; 143 take social studies; 241 take biology; 300 take history; 213 take algebra and language arts; 264 take algebra and social studies; 144 take algebra and biology; 121 take algebra and history; 111 take language arts and social studies; 90 take language arts and biology; 80 take language arts and history; 60 take social studies and biology; 70 take social studies and history; 60 take biology and history; 50 take algebra, language arts, and social studies; 50 take algebra, language arts, and biology; 50 take algebra, language arts, and history; 50 take algebra, social studies, and biology; 50 take algebra, social studies, and history; 50 take algebra, biology, and history; 50 take language arts, social studies, and biology; 50 take language arts, social studies, and history; 50 take language arts, biology, and history; 50 take social studies, biology, and history; 20 take algebra, language arts, social studies, and biology; 15 take algebra, language arts, social studies, and history; 15 take algebra, language arts, biology, and history; 10 take algebra, social studies, biology, and history; 10 take language arts, social studies, biology, and history; 5 take all five. None take none;

How many students attend Albertine’s school?

1. A $7 \times 1$ board is completely covered by $m \times 1$ tiles without overlap; each tile may cover any number of consecutive squares, and each tile lies completely on the board. Each tile is either red, blue, or green. Let $N$ be the number of tilings of the $7 \times 1$ board in which all three colors are used at least once. For example, a $1 \times 1$ red tile followed by a $2 \times 1$ green tile, a $1 \times 1$ green tile, a $2 \times 1$ blue tile, and a $1 \times 1$ green tile is a valid tiling. Note that if the $2 \times 1$ blue tile is replaced by two $1 \times 1$ blue tiles, this results in a different tiling. Find the remainder when $N$ is divided by $1000$;
2. Let F(n) denote the nth Fibonacci number (assuming that F(1) = F(2) = 1).

Prove that = F(n)F(n+1) for all n 3

1. Let G(n) denote a generalized Fibonacci number defined by G(1) = a, G(2) = b, and G(n+2) = G(n+1) + G(n) for n 1.

Prove that G(n) = a F(n-2) + b F(n-1) for all n 2.

5.