# Math 201 TEST 3-B Take-home 20 October 2015

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Instructions:** This take-home test is due on ***Tuesday, December 1st, at 5 pm***. You may speak with other students about these problems, consult other texts, or the web as long as you *acknowledge your sources when you turn in the test.*



*Answer any 3 of the following 6 problems. (You will be given extra credit for answering more than 3 problems.)*

*To receive credit, you must show your work!*

1. How many positive integers, *n*, exist such that n2 is less than 106 and a multiple of 24?
2. Prove that log2 3 is irrational.
3. Let *a, b, c, m* be integers and *m* non-zero. If gcd(a, m) = 1 and gcd(b, m) = 1, prove that gcd(ab, m) = 1
4. Let *x, y* ∈ Z, and *p* be prime. Prove *(the algebra student’s dream):*

(x + y)p ≡ xp + yp (mod *p*)

1. Show that x2 + 2y2 = 8z + 5 has no integer solution.
2. Let P(x) be a polynomial such that when P(x) is divided by x – 19, the remainder is 99, and when P(x) is divided by x – 99, the remainder is 19. What is the *remainder* when P(x) is divided by (x – 19) (x – 99)?