

Name: _____

INSTRUCTIONS: This take-home test is due on *Tuesday, December 1st, at 5 pm*. You may speak with other students about these problems, consult other texts, or the web as long as you *acknowledge your sources when you turn in the test*.



Answer any 3 of the following 6 problems. (You will be given extra credit for answering more than 3 problems.)

To receive credit, you must show your work!

1. How many positive integers, n , exist such that n^2 is less than 10^6 and a multiple of 24?
2. Prove that $\log_2 3$ is irrational.
3. Let a, b, c, m be integers and m non-zero. If $\gcd(a, m) = 1$ and $\gcd(b, m) = 1$, prove that $\gcd(ab, m) = 1$
4. Let $x, y \in \mathbb{Z}$, and p be prime. Prove (*the algebra student's dream*):
$$(x + y)^p \equiv x^p + y^p \pmod{p}$$
5. Show that $x^2 + 2y^2 = 8z + 5$ has no integer solution.
6. Let $P(x)$ be a polynomial such that when $P(x)$ is divided by $x - 19$, the remainder is 99, and when $P(x)$ is divided by $x - 99$, the remainder is 19. What is the *remainder* when $P(x)$ is divided by $(x - 19)(x - 99)$?