# Class discussion: Cardinality

**9 November 2017**



[M. C. Escher](http://www.mcescher.com/) strived to represent

artistically the concept of infinity

**I** What does it mean to say that two sets have the *same cardinality*? What does it mean to say that a set is *countable*?

**II** Show that each of the following sets is countable:

1. The set of non-negative integers.

(b)     The set of integers greater than or equal to 13.

(c)    **Z**

(d)    The set of positive even integers.

(e)     The set of even integers.

(f)     The set of odd integers.

(g) The set of rational numbers strictly between 0 and 1.

**III** (a) Show that a subset of a countable set is either finite or countable.

(b) Show that if *A* and *B* are disjoint countable sets then so is the union of *A* and *B*. What if *A* and *B* are not disjoint?

(c) Show that if *A* and *B* are countable sets then so is the Cartesian product of *A* and *B*.

(d) Prove that a countable union of countable sets is countable.

(e) Prove that the set of rational numbers strictly between 0 and 1.

(f) Demonstrate that **Q** is countable.

**IV** Show that if S is a collection of sets, then cardinality is an equivalence relation on S.

**V** Using Cantor’s diagonal argument, prove that **R** is not countable.

**VI** (a) Let *X* be a set. Define the power set, *P(X)*, of *X*.

1. Show that the power set of a finite set is finite. In such case, describe the relationship between |X| and |P(X)|.
2. Prove *Cantor’s Theorem*: X and P(X) are not of the same cardinality.

**Highly recommended:**

MIT lecture notes on cardinality, 24.118 (paradox and infinity)



[Georg Ferdinand Ludwig Cantor](http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Cantor.html) (1845 – 1918) is best known for

his discovery of transfinite numbers and the creation of Set Theory.

[Course Home Page](http://www.math.luc.edu/~ajs/courses/201fall2017/index.pdf) [Department Home Page](http://www.math.luc.edu/) [Loyola Home Page](http://www.luc.edu/)