MATH 201:   CLASS DISCUSSION   (31 AUG 2017)

NAÏVE SET THEORY

STUDY CAREFULLY SECTIONS 1.1 – 1.7 OF HAMMACK

A. List the elements of each of the following sets: (a) $\{x \in R : x^4 - 1 = 0\}$ (b) $\{x \in Z : -1/3 < x < 5.99\}$

   (c) $\{x \in N \mid x \le 4\}$ (d) {unicorns| unicorn lives in Illinois}   (e) $\{\}$ (f) $\{\varphi\}$

   (g) $\{1, \{2\}\}$

B. Write in set notation: (a) $\{4, 9, 16, 25, \ldots\}$ (b) $\{1/1, 1/3, 1/5, 1/7, \ldots\}$ (c) $\{\ldots 1/8, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

C. Determine the cardinality of each set in (1).

D. Find each of the following cardinalities:

$$|\{\{1\}, \{2, \{3,4\}\}, \varnothing\}|$$
$$|\{\{1,4\}, a, b, \{\{3,4\}\}, \{\varnothing\}\}|$$
$$|\{\{\{1\}, \{2, \{3,4\}\}, \varnothing\}\}|$$
$$|\{\{\{1,4\}, a, b, \{\{3,4\}\}, \{\varnothing\}\}\}|$$
$$|\{x \in Z : |x| < 10\}|$$

E. Sketch the following sets of points in the xy-plane.

$$\{(x,y) : x, y \in \mathbb{R}, x^2 + y^2 \le 1\}$$
$$\{(x,y) : x, y \in \mathbb{R}, y \ge x^2 - 1\}$$
$$\{(x,y) : x, y \in \mathbb{R}, x > 1\}$$
$$\{(x, x+y) : x \in \mathbb{R}, y \in \mathbb{Z}\}$$
$$\{(x, \tfrac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\}$$
$$\{(x,y) \in \mathbb{R}^2 : (y-x)(y+x) = 0\}$$
$$\{(x,y) \in \mathbb{R}^2 : (y-x^2)(y+x^2) = 0\}$$

F.

1. Suppose $A = \{1,2,3,4\}$ and $B = \{a,c\}$.

(a) $A \times B$	(c) $A \times A$	(e) $\varnothing \times B$	(g) $A \times (B \times B)$
(b) $B \times A$	(d) $B \times B$	(f) $(A \times B) \times B$	(h) B^3

2. Suppose $A = \{\pi, e, 0\}$ and $B = \{0,1\}$.

(a) $A \times B$	(c) $A \times A$	(e) $A \times \varnothing$	(g) $A \times (B \times B)$
(b) $B \times A$	(d) $B \times B$	(f) $(A \times B) \times B$	(h) $A \times B \times B$

3. $\{x \in \mathbb{R} : x^2 = 2\} \times \{a, c, e\}$ **6.** $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\}$
4. $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\}$ **7.** $\{\varnothing\} \times \{0, \varnothing\} \times \{0,1\}$
5. $\{x \in \mathbb{R} : x^2 = 2\} \times \{x \in \mathbb{R} : |x| = 2\}$ **8.** $\{0,1\}^4$

Sketch these Cartesian products on the x-y plane $\mathbb{R}^2$ (or $\mathbb{R}^3$ for the last two).

9. $\{1,2,3\} \times \{-1,0,1\}$ **15.** $\{1\} \times [0,1]$
10. $\{-1,0,1\} \times \{1,2,3\}$ **16.** $[0,1] \times \{1\}$
11. $[0,1] \times [0,1]$ **17.** $\mathbb{N} \times \mathbb{Z}$
12. $[-1,1] \times [1,2]$ **18.** $\mathbb{Z} \times \mathbb{Z}$
13. $\{1,1.5,2\} \times [1,2]$ **19.** $[0,1] \times [0,1] \times [0,1]$
14. $[1,2] \times \{1,1.5,2\}$ **20.** $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\} \times [0,1]$

G. Find the power set of each of the following sets:

1. $\{1,2,3,4\}$ 5. $\{\emptyset\}$

2. $\{1,2,\emptyset\}$ 6. $\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$

3. $\{\{\mathbb{R}\}\}$ 7. $\{\mathbb{R},\{\mathbb{Q},\mathbb{N}\}\}$

4. $\emptyset$ 8. $\{\{0,1\},\{0,1,\{2\}\},\{0\}\}$

H.

Write out the following sets by listing their elements between braces.

9. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 2\}$ 11. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 4\}$

10. $\{X \subseteq \mathbb{N} : |X| \le 1\}$ 12. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 1\}$

Decide if the following statements are true or false. Explain.

13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$ 15. $\{(x,y) : x - 1 = 0\} \subseteq \{(x,y) : x^2 - x = 0\}$

14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$ 16. $\{(x,y) : x^2 - x = 0\} \subseteq \{(x,y) : x - 1 = 0\}$

I.

Find the indicated sets.

1. $\mathscr{P}(\{\{a,b\},\{c\}\})$ 7. $\mathscr{P}(\{a,b\}) \times \mathscr{P}(\{0,1\})$

2. $\mathscr{P}(\{1,2,3,4\})$ 8. $\mathscr{P}(\{1,2\} \times \{3\})$

3. $\mathscr{P}(\{\{\emptyset\},5\})$ 9. $\mathscr{P}(\{a,b\} \times \{0\})$

4. $\mathscr{P}(\{\mathbb{R},\mathbb{Q}\})$ 10. $\{X \in \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$

5. $\mathscr{P}(\mathscr{P}(\{2\}))$ 11. $\{X \subseteq \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$

6. $\mathscr{P}(\{1,2\}) \times \mathscr{P}(\{3\})$ 12. $\{X \in \mathscr{P}(\{1,2,3\}) : 2 \in X\}$

Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

13. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))|$ 17. $|\{X \in \mathscr{P}(A) : |X| \le 1\}|$

14. $|\mathscr{P}(\mathscr{P}(A))|$ 18. $|\mathscr{P}(A \times \mathscr{P}(B))|$

15. $|\mathscr{P}(A \times B)|$ 19. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A \times \emptyset)))|$

16. $|\mathscr{P}(A) \times \mathscr{P}(B)|$ 20. $|\{X \subseteq \mathscr{P}(A) : |X| \le 1\}|$