# Class discussion: 28 September 2017

# Inclusion/exclusion; Direct proofs

**I Inclusion/Exclusion**

1. Suppose there are 200 first-year students, 100 of whom are taking Calculus and 70 of whom are taking Algebra. If there are 50 first-year students that are taking both Calculus and Algebra, how many first-year students are taking neither course?
2. In a survey on the chewing gum preferences of soccer players, it was found that 22 like fruit, 25 like spearmint, 39 like grape, 9 like spearmint and fruit, 17 like fruit and grape, 20 like spearmint and grape, 6 like all flavors, 4 like none. How many players were surveyed?
3. How many integers between 1 and 10000 (inclusive) are divisible by 3 or 5?

How many are divisible by 3, 5 or 7?

1. Of a hundred volunteers who filled out questionnaires about their viewing habits over the past year: (i) 28 watched gymnastics (ii) 29 watched baseball (iii) 19 watched soccer (iv) 14 watched gymnastics and baseball (v) 12 watched baseball and soccer (vi) 10 watched gymnastics and soccer (vii) 8 watched all three sports. How many watched none of the three sports?
2. In a mathematics department of size 40, faculty are members of four different professional organizations: NCTM (N), MAA (M), AMS (A), and SSMA (S). We know that 21 are members of NCTM, 26 are MAA members, 19 belong to AMS, and 17 are in SSMA. In addition, we know that 15 are in both NCTM and MAA, 6 are in NCTM and AMA, 9 are in NCTM and SSMA, 14 and in MAA and AMS, 10 are in MAA and SSMA, and 11 are in AMS and SSMA. We also know that 6 are in NCTM, MAA, and AMS, 5 are in NCTM, MAA, and SSMA, 4 are in NCTM, AMS, and SSMA, and 9 are in MAA, AMS, and SSMA. Finally, 4 people belong to all four organizations. How many faculty members belong to none of these organizations?

**II** Direct proofs

1. Prove that if *a* is even, then so is a2 and if a is odd then so is a2.
2. Let n be any integer. Prove that n2 + 3n + 4 is even. Hint: consider cases.
3. Prove that if $n\in N, then 1+\left(-1\right)^{n}\left(2n-1\right)is a multiple of 4.$
4. Prove that every multiple of 4 can be written as $1+\left(-1\right)^{n}\left(2n-1\right) $for some$ n\in N$.
5. Prove that if two integers have opposite parity, then their sum is odd.
6. Prove that if *n* is the form 3K + 1, then n2 is of the form 3L + 1.
7. Prove that if *n* is the form 5K + 3, then n2 is of the form 5L + 4.
8. Let n be larger than 6. Prove that n2 – 25 cannot be prime.
9. Let n be an integer larger than 3. Then n3 – 8 cannot be prime.
10. Prove that the geometric mean of any two positive real numbers is less than or equal to its arithmetic mean. Hint: Begin with (a – b)2 ≥ 0.
11. Prove that if n is odd, then n3 is odd.
12. Prove that if two integers have opposite parity, then their product is even.
13. Let a, b, c be integers. Prove that if a2 | b and b3|c, then a6|c.
14. Let n be an integer. Prove that if 5|2a then 5|a.
15. Let n be an integer. Prove that if 7|4a then 7f|a.
16. Let x and y be real numbers. Prove that if x2 + 5y = y2 + 5x. Prove that either x = y or x+y=5.

*The theory of Numbers has always been regarded as one of the most obviously useless branches of Pure Mathematics. The accusation is one against which there is no valid defense; and it is never more just than when directed against the parts of the theory which are more particularly concerned with primes. A science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life. The theory of prime numbers satisfies no such criteria. Those who pursue it will, if they are wise, make no attempt to justify their interest in a subject so trivial and so remote, and will console themselves with the thought that the greatest mathematicians of all ages have found it in it a mysterious attraction impossible to resist.*

                                                             - G. H. Hardy from a 1915 lecture on prime numbers

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