

MATH 201: CLASS DISCUSSION (5 SEPT 2017)

NAÏVE SET THEORY CONTINUED

INTRO TO PROOFS

1. Let A , B and C be three sets such that:

Set $A = \{2, 4, 6, 8, 10, 12\}$, set $B = \{3, 6, 9, 12, 15\}$ and set $C = \{1, 4, 7, 10, 13, 16\}$.

Find:

(i) $A \cup B$

(ii) $A \cap B$

(iii) $B \cap A$

(iv) $B \cup A$

(v) $B \cup C$

(vi) $A - B$

(vii) $A - (B \cup C)$

(viii) $A - (B \cap C)$

(ix) Is $A \cup B = B \cup A$?

(x) Is $B \cap C = B \cup C$?

2. Complete each of the following:

(i) **Associativity of set union and intersection:**

$$A \cup (B \cup C) =$$

$$A \cap (B \cap C) =$$

(ii) **Commutativity:** $A \cup B =$

$$A \cap B =$$

(iii) **Distributivity:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$

$$A \cap (B \cup C) =$$

(iv) **De Morgan Laws:** $(A \cup B)^c =$

$$(A \cap B)^c =$$

(v) **Complementation:** $A \cup A^c =$

$$A \cap A^c =$$

(vi) **Double complement:** $(A^c)^c =$

3. True or False? Give proof or counterexample.

(a) $A \cup B \subseteq A \cap B$

(b) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(c) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

(d) $A - (B \cap C) = (A - B) \cup (A - C)$

(e) $A - B = B^c - A^c$

(f) $(A \cup B) \cap C \supseteq (A \cup B) \cap (A \cup C)$

4. [Halmos, Naïve Set Theory]

EXERCISE. A necessary and sufficient condition that $(A \cap B) \cup C = A \cap (B \cup C)$ is that $C \subset A$. Observe that the condition has nothing to do with the set B .

5. [Halmos, Naïve Set Theory]

(a) Prove that $P(E) \cap P(F) = P(E \cap F)$

(b) Prove that $P(E) \cup P(F) \subseteq P(E \cup F)$

