**MATH 201 Practice Problems for Test III**

 **Part I**

1. Let X be a set. Define P(X), the *power set* of X.
2. Let F: X →Y be a mapping. Define: F is *injective*.
3. Let G: X →Y be a mapping. Define: G is *surjective*.
4. Let S be a set. Define: S is *countably infinite*.
5. Let R and S be sets. What does it mean to assert that R and S *are of the same cardinality*?
6. If |A| = 9876, then |P(A)| =
7. Let X and Y be sets and let G: X→Y be a bijection. Define G-1: Y → X
8. 1599999 (mod 16) is congruent to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
9. If a ≡ 9 (mod 11) and b ≡ 7 (mod 11), then 3a + 5b ≡ \_\_\_\_\_\_\_\_\_\_\_\_\_ (mod 11)
10. Let S be a set. Then an equivalence relation on S corresponds to a \_\_\_\_\_\_\_\_\_\_\_\_ on S and vice-versa.

**Part II**

1. *Here is a partial proof of Cantor’s theorem. Your job is to complete it.*

**Theorem:** For any set X, P(X) and X are not of the same cardinality.

Proof: Let X be a set and P(X) be the power set of X.

*Strategy:*  proof by contradiction.

Assume that ∃ bijection F: X →P(X).

Define $D^{\*}= \left\{q\in X \left|q\notin F(q\right.)\right\}$

Then:

1. Odette believes that the set of all *finite* sequences using the alphabet {a, b, c} is countably infinite. Give proof or counterexample. (For example: aab, cabcab, ccccccccccccccc are all finite sequences.)
2. Swann believes that the set of all *infinite* sequences using the alphabet {a, b, c} is countably infinite. Give proof or counterexample. (For example: abababab… , abcabccccbbaabbbbb… are infinite strings.)
3. Let A, B be non-empty finite sets. Suppose that |A|=1789 and |B| = 2016. Then
4. The number of surjections from A into B is \_\_\_\_\_\_\_\_\_\_\_. Why?
5. The number of injections from A into B is \_\_\_\_\_\_\_\_\_\_\_. Why?

5. Let Q be the set of all rational numbers. Define f: Q → Q as follows:



(a) Is *f* well-defined? Justify your answer.

(b) Is *f* injective? Why?

(c) Is *f* surjective? Why?

6. For any real numbers, *c* and *d*, let us define the binary operation  as follows:

c  d = c2 + d2 – 1

Give either a *brief* justification or counterexample for each of the following assertions:

1. The set of integers is closed under the operation  .
2. The set of even integers is closed under the operation  ?
3. The set of odd integers is closed under .
4. The set of positive integers is closed under .
5. The set of rational numbers is closed under .
6. The set of irrational numbers is closed under .

7. Let X = {0, 2, 5, 8, 13, 15} and Y = P(X). Define T: X →P(X) as follows:



 Find D\* (as defined in Part II, problem

8. Give an *example* of three sets, X, Y, S, and two mappings, F: X →Y, G: Y →S such that $G$ is *not* injective yet $G∘F$ is injective.

9. Let S be the set of all polynomials for which odd powers of x (i.e., 1, 3, 5, 7,…) do not appear.

For example, 4 + ½ x2 –  x8 $\in S, but x^{2}-\frac{5}{4}x^{81} \notin S$.

(a) Is S closed under differentiation? Why?

1. Is S closed under the operation of taking the second derivative?
2. Is S closed under multiplication by x3?
3. Is S closed under multiplication by x4?

(e) Is S closed under the following operation \*:

 For all $p\in S p\* = $p$(1+ x^{4})$

(f) Is S closed under the following operation ☹:

 For all $p\in S p$☹ = p$(1+x)$

10. Define the following binary relation R on **Z+** : For c, d ∈ **Z+**, cRd if |c – d | < 5. *(Justify each answer!)*

 (a) Is R reflexive? Why?

 (b) Is R symmetric? Why?

 (c) Is R transitive? Why?

11. Define the binary relation R on a non-empty set, S, of books as follows:

 For a, b ∈ S, aRb if book ***a*** costs more **and** contains fewer pages than book ***b***. *(Justify each answer!)*

 (a) Is R reflexive? Why?

 (b) Is R symmetric? Why?

 (c) Is R transitive? Why?

12. Again, let S be a non-empty set of books. Let H be the binary relation defined by:

 for *a*, *b* ∈ S, aHb if book ***a*** costs more ***or*** contains fewer pages than book ***b***. *(Justify each answer!)*

(a) Is H reflexive? Why?

 (b) Is H symmetric? Why?

 (c) Is H transitive? Why?