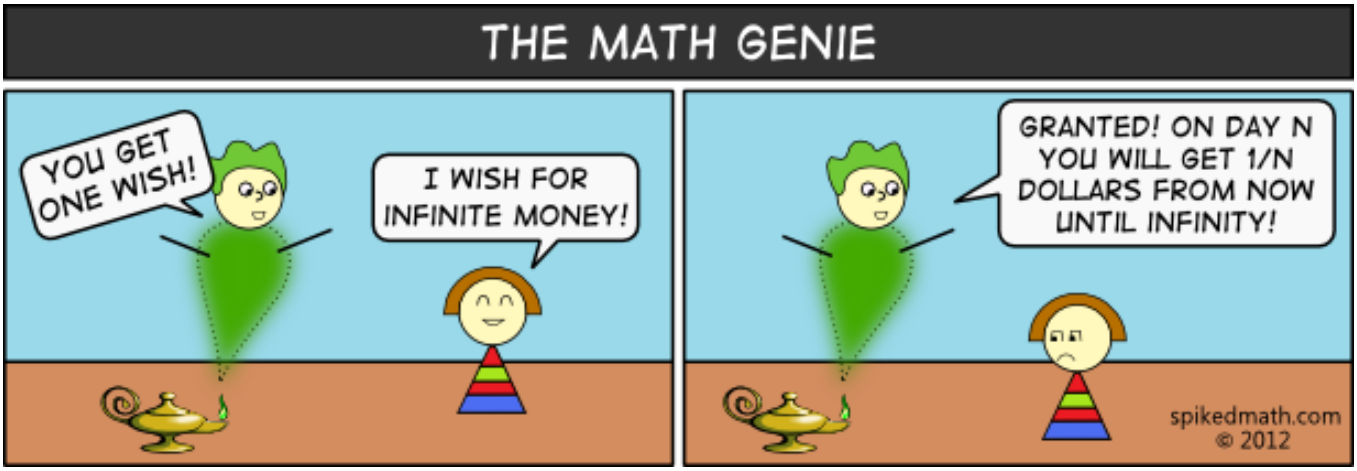
MATH 351: QUESTIONS FOR DISCUSSION, 31 AUGUST 2018



Read (and reread) Chapter 1 of Mattuck (and answer all of the questions in the chapter). Review induction; review the proof of the binomial theorem.

1. Prove (without using calculus) that the sequence
$$c_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$
is not bounded above.

2. Let *n* be a non-negative integer. Explain why $1 + r + r^2 + r^3 + \cdots + r^n = \frac{1-r^{n+1}}{1-r}$. What condition must be placed upon *r*?

3. Prove that the sequence $\left\{\left(1 + \frac{1}{2^n}\right)^{2^n}\right\}$ *is increasing*. Hint: Begin with $(1+b)^2 > 1+2b$ for $b \neq 0$. Then raise each side to the power 2^n.

4. State the binomial theorem for $(1 + x)^n$ where *n* is a positive integer.
Prove the binomial theorem using induction. Also, give a combinatorial argument for the binomial theorem. Prove that the sequence $\left\{\left(1 + \frac{1}{k}\right)^k\right\}$ is bounded above. Can we conclude that the sequence $\left\{\left(1 + \frac{1}{2^k}\right)^{2^k}\right\}$ converges? Why?

5. What does n>>1 mean? Prove that if $\{a_n\}$ and $\{b_n\}$ are increasing sequences for n>>1, then $\{a_n + b_n\}$ is increasing for n>>1.

6. What is the **Completeness Property** of **R**?

7. What is an **Equivalence Relation** on a set X?

Review exercises on equivalence relations:

Let R be a relation on a set S. What does it mean for R to be *reflexive*? *symmetric*? *transitive*? What is an *equivalence relation* on S? Explain how an equivalence relation corresponds to a partition on the set S. What does the term *equivalence class* mean?

(A) Determine which of the three properties "reflexive," "symmetric," and "transitive," apply to each of the following relations on the set of integers. For each relation that is an equivalence relation, describe the equivalence classes.

a R b iff

1. $a = b$
2. $a \leq b$
3. $a < b$
4. $a \mid b$
5. $|a| = |b|$
6. $a^2 + a = b^2 + b$
7. $a < |b|$
8. $ab > 0$
9. $ab \geq 0$
10. $a + b > 0$
11. $a \equiv b \mod 4$
12. $a \equiv b \mod m$ (where $m > 0$)

(B) Do the same as in (A) for the following relations on the set of all people. p R q iff

a. p "is a father of" q

b. p "is a sister of" q

c. p "is a friend of" q

d. p "is an aunt of" q

e. p "is a descendant of" q

f. p "has the same height" as q

g. p "likes" q

h. p "knows" q

i. p "is married to" q