Math 351: Questions for class discussion, 3rd December

Open & closed sets; compactness

**Metric Spaces & Compactness:** Rudin (2nd edition) , pp. 27 – 35

1. *(Review)*  Define **Metric Space**.
2. Show that each of the following defines a metric on the given space:





1. In information theory, linguistics and computer science, the **Levenshtein distance** is a string metric for measuring the difference between two sequences. Informally, the Levenshtein distance between two words is the minimum number of single-character edits (insertions, deletions or substitutions) required to change one word into the other. It is named after the Soviet mathematician Vladimir Levenshtein, who considered this distance in 1965. Levenshtein distance may also be referred to as **edit distance**, although that term may also denote a larger family of distance metrics.[[2]](https://en.wikipedia.org/wiki/Levenshtein_distance#cite_note-navarro-2):32 It is closely related to pairwise string alignments.
2. Let X = **R**2 and let d(x, y) = ||x||+ ||y|| be the sum of the Euclidean distances of x and y from the origin, unless x and y lie on the same line through the origin, in which case it is the Euclidean distance from x to y. In Britain, *d* is sometimes called the “British Rail” metric, because all the train lines radiate from London (located at the origin). To take a train from town x to town y, one has to take a train from x to 0 and then take a train from 0 to y, unless x and y are on the same line, in which case one can take a direct train.
3. *(Review)*  Define *open subset* of *X.*
4. Let (X, d) be a metric space and let Nr(p) represent the *neighborhood of p with radius r*, *viz*

 $N\_{r}\left(p\right)=\left\{x\in X\right|d\left(x,p\right)<r\}. $Prove that $N\_{r}\left(p\right)$ is open.

1. Define *isolated point*.
2. Let A = $\left\{\frac{1}{n}:n\in N\right\}. Identify the isolated points of A.$
3. *(Review)*  Define *limit point* of a subset, S, of a metric space.
4. *What are the limit points of* ***Q****?*
5. *(Review)*  Define *closed subset* of *X*.
6. Prove that a subset *S* of a metric space is open if and only if its complement Sc is closed.
7. Show that the Cantor set is closed.
8. Is the union of a finite number of open sets open? Is the union of a finite number of closed sets closed?
9. Is the intersection of a finite number of open sets open? Is the intersection of a finite number of closed sets closed?
10. Answer questions 12 and 13 omitting the hypotheses that there are only a finite number of sets.
11. *Let (X, d) be a metric space. Let Y be any non-empty subset of* X. Explain why *Y* is a metric space with the “inherited” metric.
12. Give an example of a countable collection of open sets in **R** whose intersection is closed, non-empty, and not all of **R**.

Consider **R** with the usual metric. Decide whether each of the following subsets of **R** are open, closed, both, or neither. If a set is not open, find a point in the set for which there is no neighborhood contained in the set. IF a set is not closed, find a limit point that is not contained in the set.

1. **Q** (b) **R** (c) the empty set (d) $\left\{x\in R\right| x\ne 0\}$ (e) $\left\{1+\frac{1}{4}+\frac{1}{9}+…+\frac{1}{n^{2}}\right| n\in N\}$

(f)$\left\{x\in Q\right| 0<x<1\}$ (g) $\left\{(-1)^{n}+\frac{2}{n}\right| n\in N\}$

1. Define **Open Cover** of a set S.
2. Give an example of an open cover of (0, 1) that has no finite subcover.
3. Define compact set in the context of a metric space (X, d).
4. Is the intersection of a set of compact sets compact? Is the union of a set of compact sets compact? Specify if the answer is true for a finite number of sets, countably many sets, uncountably many sets.
5. Prove that every compact subset of a metric space is closed.
6. State and prove the Heine-Borel theorem.

