Math 351: Questions for class discussion, 5th December

t-Compactness; Heine-Borel Theorem

Rudin (2nd edition) , chapter 2

1. Let A = $\left\{\frac{1}{n}:n\in N\right\}. Identify the isolated points of A.$
2. *What are the limit points of* ***Q****?*
3. Prove that a subset *S* of a metric space is open if and only if its complement Sc is closed.
4. Show that the Cantor set is closed.
5. Is the union of a finite number of open sets open? Is the union of a finite number of closed sets closed?
6. Is the intersection of a finite number of open sets open? Is the intersection of a finite number of closed sets closed?
7. Answer questions 5 and 6 omitting the hypotheses that there are only a finite number of sets.
8. *Let (X, d) be a metric space. Let Y be any non-empty subset of* X. Explain why *Y* is a metric space with the “inherited” metric.
9. Give an example of a countable collection of open sets in **R** whose intersection is closed, non-empty, and not all of **R**.
10. Consider **R** with the usual metric. Decide whether each of the following subsets of ***R*** are open, closed, both, or neither. If a set is not open, find a point in the set for which there is no neighborhood contained in the set. If a set is not closed, find a limit point that is not contained in the set.
11. **Q** (b) **R** (c) the empty set (d) $\left\{x\in R\right| x\ne 0\}$ (e) $\left\{1+\frac{1}{4}+\frac{1}{9}+…+\frac{1}{n^{2}}\right| n\in N\}$

(f)$\left\{x\in Q\right| 0<x<1\}$ (g) $\left\{(-1)^{n}+\frac{2}{n}\right| n\in N\}$

1. Define **Open Cover** of a set S. Give an example of an open cover of (0, 1) that has no finite subcover.
2. Define compact (aka t-compact) set in the context of a metric space (X, d).
3. Is the intersection of a set of compact sets compact? Is the union of a set of compact sets compact? Specify if the answer is true for a finite number of sets, countably many sets, uncountably many sets.
4. Prove that every compact subset of a metric space is closed.
5. Prove that every compact subset is bounded.
6. Show that if K is copact and non-empty, then sup K and inf K both exist and are elements of K.
7. State and prove the Heine-Borel theorem.
8. Decude wgucg of the following sets are copact subsets of R.
9. N (b) $Q∩[0,1]$ (c) the Cantor set (d) $\left\{1+\frac{1}{4}+\frac{1}{9}+…+\frac{1}{n^{2}}\right| n\in N\}$

(e) $\{1,\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5}, …\}$

1. True or False? (a) The arbitrary intersection of compact sets is compact.
2. The arbitrary union of compact sets is compact.
3. Let A be arbitrary, and let K be compact. Then, the intersection of A and K is compact.
4. $If F\_{1}⊇F\_{2}⊇F\_{3}⊇F\_{4}⊇…$ is a nested sequence of non-empty closed sets, then the intersection $\bigcap\_{1}^{\infty }F\_{n} \ne ∅.$

