Math 351: Questions for class discussion, 5th December

t-Compactness; Heine-Borel Theorem

Rudin (2nd edition) , chapter 2

1. Let A =
2. *What are the limit points of* ***Q****?*
3. Prove that a subset *S* of a metric space is open if and only if its complement Sc is closed.
4. Show that the Cantor set is closed.
5. Is the union of a finite number of open sets open? Is the union of a finite number of closed sets closed?
6. Is the intersection of a finite number of open sets open? Is the intersection of a finite number of closed sets closed?
7. Answer questions 5 and 6 omitting the hypotheses that there are only a finite number of sets.
8. *Let (X, d) be a metric space. Let Y be any non-empty subset of* X. Explain why *Y* is a metric space with the “inherited” metric.
9. Give an example of a countable collection of open sets in **R** whose intersection is closed, non-empty, and not all of **R**.
10. Consider **R** with the usual metric. Decide whether each of the following subsets of ***R*** are open, closed, both, or neither. If a set is not open, find a point in the set for which there is no neighborhood contained in the set. If a set is not closed, find a limit point that is not contained in the set.
11. **Q** (b) **R** (c) the empty set (d) (e)

(f) (g)

1. Define **Open Cover** of a set S. Give an example of an open cover of (0, 1) that has no finite subcover.
2. Define compact (aka t-compact) set in the context of a metric space (X, d).
3. Is the intersection of a set of compact sets compact? Is the union of a set of compact sets compact? Specify if the answer is true for a finite number of sets, countably many sets, uncountably many sets.
4. Prove that every compact subset of a metric space is closed.
5. Prove that every compact subset is bounded.
6. Show that if K is copact and non-empty, then sup K and inf K both exist and are elements of K.
7. State and prove the Heine-Borel theorem.
8. Decude wgucg of the following sets are copact subsets of R.
9. N (b) (c) the Cantor set (d)

(e)

1. True or False? (a) The arbitrary intersection of compact sets is compact.
2. The arbitrary union of compact sets is compact.
3. Let A be arbitrary, and let K be compact. Then, the intersection of A and K is compact.
4. is a nested sequence of non-empty closed sets, then the intersection

