Math 351: Questions for class discussion, 12th November

Bolzano’s theorem; Intermediate value theorem



1. *(Review)* Prove that, for any real numbers *a* and *b*, .

What is the corresponding statement for min{a, b}?

1. Prove that if f is continuous then |f(x)| is continuous as well.
2. *(Review)* Find four continuous functions y = f(x) satisfying y2 = x2.
3. *(Review)* Write the negation of the statement: f(x) is continuous at x = p
4. *(Review)* Let

Prove, using the Sequential Continuity criterion, that f(x) is discontinuous at x = 0.

1. Prove the Positivity Theorem for continiuous functions.
2. [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)] Let *f* be a function defined on **R**.
3. Let’s say f is *onetinuous* at c if for all

whenever . Find an example of a function that is *onetinuous* on all of R.

1. Let’s say f is *equaltinuous* at c if for all

whenever . Find an example of a function that is *equaltinuous* on R but is nowhere *onetinuous*, or explain why there is no such function.

1. Let’s say f is *lesstinuous* at c if for all can choose and it follows that

 whenever. Find an example of a function that is *lesstinuous* on **R** that is nowhere equaltinuous, or explain why there is no such function.

1. Is every *lesstinuous* function continuous? Is every continuous function *lesstinuous*? Explain.
2. Assume that f and g are defined on all of **R**, and that .

Give a counterexample to

1. State and prove Bolzano’s theorem for a continuous function on a compact interval.
2. Show that the Intermediate Value Theorem is a consequence of Bolzano’s theorem.
3. State the Intersection Principle.
4. Apply the intermediate value theorem to show that the equation has a solution in the interval [0, 1].
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6. Prove that f(x) = 9 sin x – x5 = 1 has at least one solution.
7. State the Intersection Principle.
8. State the Intermediate Value Property.
9. Prove that if f(x) is strictly monotone and has the IVP on [a, b], then f is continuous on [a, b].
10. State and sketch the proof of the Inverse Function Theorem.



1. Define sequentially compact for a subset of **R**.
2. Prove the Sequential Compactness theorem.