MATH 351: QUESTIONS FOR CLASS DISCUSSION, 12TH NOVEMBER

BOLZANO'S THEOREM; INTERMEDIATE VALUE THEOREM

1. *(Review)* Prove that, for any real numbers *a* and *b*, $\max\{a, b\} = \frac{a+b+|a-b|}{2}$.

What is the corresponding statement for min{a, b}?

2. Prove that if f is continuous then |f(x)| is continuous as well.

3. *(Review)* Find four continuous functions y = f(x) satisfying $y^2 = x^2$.

4. *(Review)* Write the negation of the statement: f(x) is continuous at x = p

5. *(Review)* Let $f(x) = \begin{cases} 1 & if\ x = \frac{1}{n}\ for\ \ n \in N \\ 0 & otherwise \end{cases}$

Prove, using the Sequential Continuity criterion, that f(x) is discontinuous at x = 0.

6. Prove the Positivity Theorem for continiuous functions.

7. [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)] Let f be a function defined on **R**.

(a) Let's say f is *onetinuous* at c if for all $\varepsilon > 0\ we\ can\ choose\ \delta = 1\ and\ it\ follows\ that\ |f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$. Find an example of a function that is *onetinuous* on all of R.

(b) Let's say f is *equaltinuous* at c if for all $\varepsilon > 0\ we\ can\ choose\ \delta = \varepsilon\ and\ it\ follows\ that\ |f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$. Find an example of a function that is *equaltinuous* on R but is nowhere *onetinuous*, or explain why there is no such function.

(c) Let's say f is *lesstinuous* at c if for all $\varepsilon > 0$ *we* can choose $0 < \delta < \varepsilon$ and it follows that $|f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$. Find an example of a function that is *lesstinuous* on **R** that is nowhere equaltinuous, or explain why there is no such function.

(d) Is every *lesstinuous* function continuous? Is every continuous function *lesstinuous*? Explain.

8. Assume that f and g are defined on all of **R**, and that $\lim_{x \to p} f(x) = q\ and\ \lim_{x \to q} g(x) = r$.

Give a counterexample to $\lim_{x \to p} g(f(x)) = r$

9. State and prove Bolzano's theorem for a continuous function on a compact interval.

10. Show that the Intermediate Value Theorem is a consequence of Bolzano's theorem.

11. State the Intersection Principle.

12. Apply the intermediate value theorem to show that the equation $x^5 - 3x^2 = -1$ has a solution in the interval [0, 1].

13. Apply the intermediate value theorem to show that the equation $\sqrt{x^6 + 5x^4 + 9} = 3.5$ has a solution in the interval [0, 1].

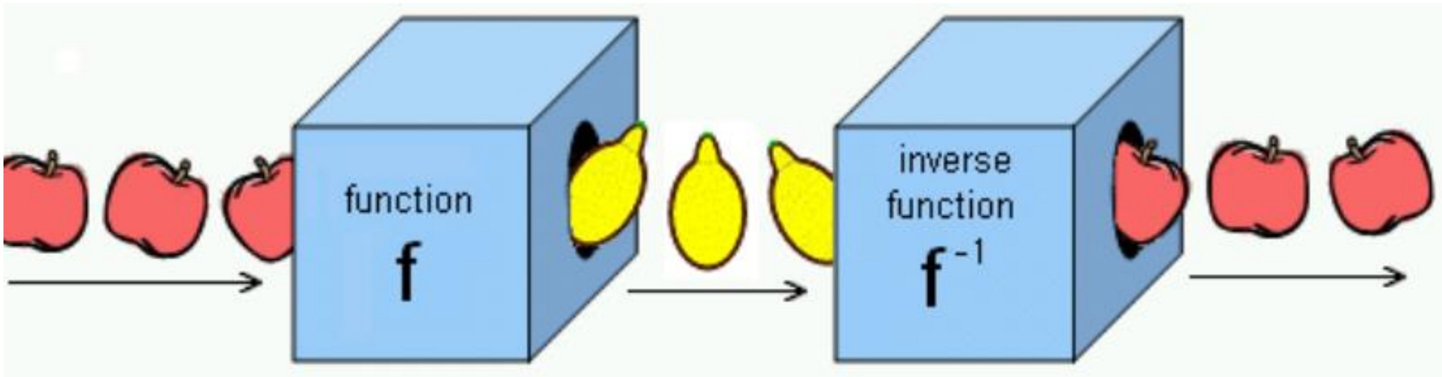
14. Prove that $f(x) = 9 \sin x - x^5 = 1$ has at least one solution.

15. State the Intersection Principle.

16. State the Intermediate Value Property.

17. Prove that if f(x) is strictly monotone and has the IVP on [a, b], then f is continuous on [a, b].

18. State and sketch the proof of the Inverse Function Theorem.



19. Define sequentially compact for a subset of **R**.

20. Prove the Sequential Compactness theorem.