Math 351: Questions for class discussion, 14th November

Compactness; extreme value theorem

1. *(Review)* Find four continuous functions y = f(x) satisfying y2 = x2.
2. *(Review)* Let

Prove, using the Sequential Continuity criterion, that f(x) is discontinuous at x = 0.

1. *(Review)* [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

Let *f* be a function defined on **R**.

1. Let’s say f is *onetinuous* at c if for all

whenever . Find an example of a function that is *onetinuous* on all of R.

1. Let’s say f is *equaltinuous* at c if for all

whenever . Find an example of a function that is *equaltinuous* on R but is nowhere *onetinuous*, or explain why there is no such function.

1. Let’s say f is *lesstinuous* at c if for all can choose and it follows that

whenever. Find an example of a function that is *lesstinuous* on **R** that is nowhere equaltinuous, or explain why there is no such function.

1. Is every *lesstinuous* function continuous? Is every continuous function *lesstinuous*? Explain.
2. *(Review)* Assume that f and g are defined on all of **R**, and that .

Give a counterexample to

1. *(Review)* Prove that f(x) = 9 sin x – x5 = 1 has at least one solution.
2. State the Intermediate Value Property.
3. Prove that if f(x) is strictly monotone and has the IVP on [a, b], then f is continuous on [a, b].
4. State and sketch the proof of the Inverse Function Theorem.
5. Define sequentially compact for a subset of **R**.
6. Prove the Sequential Compactness theorem.