MATH 351: QUESTIONS FOR CLASS DISCUSSION, 14TH NOVEMBER

COMPACTNESS; EXTREME VALUE THEOREM

1. *(Review)* Find four continuous functions y = f(x) satisfying $y^2 = x^2$.

2. *(Review)* Let $f(x) = \begin{cases} 1 & if\ x = \frac{1}{n}\ for\ \ n \in N \\ 0 & otherwise \end{cases}$

Prove, using the Sequential Continuity criterion, that f(x) is discontinuous at x = 0.

3. *(Review)* [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

Let f be a function defined on **R**.

(a) Let's say f is *onetinuous* at c if for all $\varepsilon > 0\ we\ can\ choose\ \delta = 1\ and\ it\ follows\ that\ |f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$. Find an example of a function that is *onetinuous* on all of R.

(b) Let's say f is *equaltinuous* at c if for all $\varepsilon > 0\ we\ can\ choose\ \delta = \varepsilon\ and\ it\ follows\ that\ |f(x) - f(c)| < \varepsilon$
whenever $|x - c| < \delta$. Find an example of a function that is *equaltinuous* on R but is nowhere *onetinuous*, or explain why there is no such function.

(c) Let's say f is *lesstinuous* at c if for all $\varepsilon > 0$ *we* can choose $0 < \delta < \varepsilon$ and it follows that $|f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$. Find an example of a function that is *lesstinuous* on **R** that is nowhere equaltinuous, or explain why there is no such function.

(d) Is every *lesstinuous* function continuous? Is every continuous function *lesstinuous*? Explain.

4. *(Review)* Assume that f and g are defined on all of **R**, and that $\lim_{x \to p} f(x) = q\ and\ \lim_{x \to q} g(x) = r$.

Give a counterexample to $\lim_{x \to p} g(f(x)) = r$

5. *(Review)* Prove that f(x) = 9 sin x – x^5 = 1 has at least one solution.

6. State the Intermediate Value Property.

7. Prove that if f(x) is strictly monotone and has the IVP on [a, b], then f is continuous on [a, b].

8. State and sketch the proof of the Inverse Function Theorem.

9. Define sequentially compact for a subset of **R**.

10. Prove the Sequential Compactness theorem.

11.