Math 351: Questions for class discussion, 16th November

Maximum theorem; continuous mapping theorem

Review for Test III

1. *(Review)* Find four continuous functions y = f(x) satisfying y2 = x2.
2. *(Review)* Let

Prove, using the Sequential Continuity criterion, that f(x) is discontinuous at x = 0.

1. *(Review)* [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

Let *f* be a function defined on **R**.

1. Let’s say f is *onetinuous* at c if for all

whenever . Find an example of a function that is *onetinuous* on all of R.

1. Let’s say f is *equaltinuous* at c if for all

whenever . Find an example of a function that is *equaltinuous* on R but is nowhere *onetinuous*, or explain why there is no such function.

1. Let’s say f is *lesstinuous* at c if for all can choose and it follows that

whenever. Find an example of a function that is *lesstinuous* on **R** that is nowhere equaltinuous, or explain why there is no such function.

1. Is every *lesstinuous* function continuous? Is every continuous function *lesstinuous*? Explain.
2. *(Review)* Assume that f and g are defined on all of **R**, and that .

Give a counterexample to

1. State the Intermediate Value Property.
2. Prove that if f(x) is strictly monotone and has the IVP on [a, b], then f is continuous on [a, b].
3. State and sketch the proof of the Inverse Function Theorem.
4. Define sequentially compact for a subset of **R**. Give examples of sets that are not sequentially compact.
5. State and prove the **Sequential Compactness theorem**.
6. State and prove the **Boundedness theorem**.
7. State and prove the **Maximum theorem**.
8. Give an example of a continuous function on (0, 1] with no max nor min on this interval, but which does not have the limit.
9. State the **Continuous mapping theorem**.

