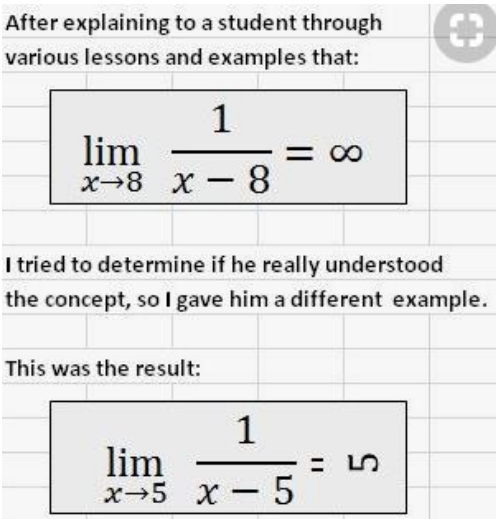
# MATH 351: QUESTIONS FOR CLASS DISCUSSION, 2ND NOVEMBER

## FUNCTIONS OF ONE VARIABLE: CONTINUITY; LIMITS; SEQUENTIAL CONTINUITY




### CONTINUITY & LIMITS

**1.** Define *function, domain, graph.*

Let f(x) be defined on (a, b) and let p ∈ $(a, b)$. Define: **f(x) is continuous at x = p.**

Give both the "Mattuck" and the traditional $\epsilon, \delta$ *definitions*.

**2.** Define: f(x) is **continuous** on (a, b).

**3.** Prove, using only the definition of continuity, that each of the following functions is continuous on the given interval.

(a) $f(x) = x^2$ on $(-\infty, \infty)$

(b) $f(x) = 1/x$ on $(0, \infty)$

(c) $f(x) = 2x^3 - 4x$ on $(-\infty, \infty)$

(d) $f(x) = \frac{x}{3+x^2}$ on $(-\infty, \infty)$

**4.** Prove that g(x) = sin x is continuous on $(-\infty, \infty)$. Hint: show that $|\sin a - \sin b| \leq |a - b|$ .

**5.** What are the four types of discontinuities?

**6.** Define: right-continuity, left-continuity. Define: **f(x) is continuous on [a, b].**

**7.** Using (4) prove that $G(x) = \int_0^{\pi} \frac{\sin xt}{t}\, dt$ *is continuous everywhere.* Which fact(s) about the Riemann integral are you taking for granted?

**8.** Let f be defined for x near *p*. Define: **the limit of f(x) as $x \to p$ exists and equals L.**

**9.** (a) Let f(x) = 3x + 1. Prove, using only the definition of limit, that

$$\lim_{x\to 2} f(x) = 7$$

(b) Let $g(x) = x^2$. Prove, using only the definition of limit, that

$$\lim_{x\to 2} g(x) = 4$$

(c) Prove that

$$\lim_{x\to 2} x^2 + x - 1 = 5$$

(d) Prove that

$$\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$$

**10.** [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

True or False? Justify!

(a) If a particular $\delta$ has been constructed as a suitable response to a particular $\varepsilon$ challenge, then any smaller positive $\delta$ will also suffice.

(b) If $\lim_{x\to b} f(x) = L$ and $b$ happens to be in the domaing of f, then L = f(b).

(c) If $\lim_{x\to b} f(x) = L, then \lim_{x\to b} 3\,(f(x) - 2)^2 = 3(L-2)^2$

(d) If $\lim_{x\to b} f(x) = 0, then \lim_{x\to b} f(x)g(x) = 0,$

for any function g $(with\ domain\ equal\ to\ the\ domain\ of\ f)$.

**11.** What is the relationship between continuity and limit? Define: limit as x $\to \infty$ $or$ x $\to -\infty$.

**12.** Prove each of the following results, using only the definition of limit.

(a) $\lim_{x\to 0} x\sin\frac{1}{x}$

(b) $\lim_{x\to 3+} \frac{|x^2-9|}{x-3}$

(c) $\lim_{x\to\infty} \frac{1}{3+x^2}$

(d) $\lim_{x\to 1} \frac{x^3-125}{x-5}$