Math 351: Questions for class discussion, 26th November

Mappings; Uniform continuity



* **Continuous Functions on Compact Intervals** (sections 3.4, 13.5)
1. *(Review)* State the **Maximum Theorem** for continuous functions on compact sets.
2. *(Review)* Explain why the Maximum Theorem implies the Boundedness Theorem.
3. *(Review)* Give an example of a continuous function on (0, 1] with neither maximum nor minimum on this interval, but which does not have the limit $\infty or-\infty as x \rightarrow 0^{+}.$
4. *(Review)* What is meant by ***the image of S*** ***under the map f*** ?
5. Find f(S) for each of the following:

(a) f(x) = x2; S = (1, 4]

(b) f(x) = ln x; S = [1, e].

(c) f(x) = sin x; S = (/4, 9/4)

(d) $f\left(x\right)=\sin(\frac{1}{x}; S=(0, 1))$

1. State and prove the **Continuous Mapping Theorem**.
2. Find functions f(x) and g(x) such that

$$\left(a\right) f(\left[-1, 1\right])=(-1, 1)$$

$$\left(b\right) g\left(\left[0, \frac{π}{2}\right]\right)=[0, \infty )$$

What general observation can you make about such functions *f* and *g*?

1. Does the Continuous Mapping Theorem imply both the IVT and the Maximum theorem?
2. (Abbott, Understanding Analysis) Give an example of each of the following, or provide a brief argument why the request is impossible.
3. A continuous function defined on [0, 1] with range (0, 1).
4. A continuous function defined on (0, 1) with range [0, 1].
5. A continuous function defined on (0, 1] with range (0, 1).
6. If f is continuous on a bounded set A, then f(A) is bounded.
7. If f is defined on R, and f(K) is compact whenever K is compact, then f is continuous on R.
8. If f is continuous on [a, b], with f(x) > 0 for all $a\leq x\leq b, then\frac{1}{f} is bounded on \left[a, b\right].$ (Meaning $\frac{1 }{f} $has bounded range.)
9. Define **uniform continuity**.
10. Prove, using only the definition, that f(x) = x2 + 1 is uniformly continuous on [0, 2] but not uniformly continuous on [0, ∞].
11. Prove that g(x) = $\sqrt{x}$ is uniformly continuous on [0, ∞).
12. Show that g(x) = x2 is *not* uniformly continuous on [0, ∞).
13. State the negation of uniform continuity (aka sequential criterion for absence of uniform convergence).
14. Show that $f\left(x\right)=\sin(\frac{1}{x} is not uniformly continuous \left(0, 1\right).)$
15. State and prove the **Uniform Continuity Theorem**.
16. Prove, using only the definition, that f(x) = x2 + 1 is uniformly continuous on [0, 2] but *not* uniformly continuous on [0, ∞].

